ON THE FIRMS’ DECISION TO HIRE ACADEMIC SCIENTISTS

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Abstract
This paper provides a theoretical rationale for private investment in basic research. It explains the decision by some firms to hire scientists who have an intrinsic motivation to pursue academic research and allow them to do so while they also dedicate time to the firm’s applied agenda. We show that this decision maximizes firms’ profits in a context where basic and applied research activities are not strong substitutes and the opportunity cost, associated with deterring scientists from remaining in academia, is sufficiently low. Allowing scientists to pursue an academic agenda facilitates participation. When scientists are privately informed about their ‘taste for science’, the contract requires that the more academically driven scientists dedicate greater attention to their personal agenda to satisfy incentive compatibility. When the reservation utility is weakly correlated with the scientist’s academic inclination, this restriction has no impact and the first best contract remains optimal. But as the correlation increases, the firms tend to select less academically driven scientists. Under-investment in basic research is not triggered by the need to reduce informational rents which are non-existent as scientists face countervailing incentives. Instead it arises from the need to curb the increased cost of efforts.

JEL: D82, D86, J31, J33, M31.

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1. INTRODUCTION

Innovation and the creation of new technologies, drugs or concepts often combine developments in abstract knowledge and applied research. Basic research and the advancement of knowledge have often been associated with universities and research centres. By contrast, the private sector has generally been perceived as a more suitable environment for applied (or proprietary) research given its potential for profit generation.\(^1\) However, these boundaries have blurred within some sectors, reflecting a changing structure of corporate research and science-industry links.

By the 1920s, large US corporations such as AT&T, General Electric, Dupont and Eastman Kodak had established in-house research groups pursuing far-sighted fundamental research alongside more applied research. These firms gradually developed internal scientific capabilities to gain a deeper understanding of the physical phenomena causing increasingly complex technical problems (Hoddeson 1981). This golden age of corporate research began to wane after the 1950s. In the 1980s and 1990s large firms became more reliant upon external inventions as they focused on incremental innovation. Universities, public research centres and research-intensive start-ups funded by venture capitalists took the lead in research in emerging fields such as biotechnology, information technologies and nanotechnology. There is a greater division of innovative labour, where large firms may still invest in scientific capabilities to be effective buyers of knowledge rather than to produce research themselves (Arora et al 2017).\(^2\)

Yet many of today’s large corporations originated as science-based start-ups and still continue to invest and produce basic research. The commercial biotechnology company Genentech, founded in 1976 by a venture capitalist and a scientist, publishes an average 200 academic publications a year.\(^3\) Google publishes hundreds of academic papers every year. The attractiveness of the company for researchers is summarised as follows by a senior research scientist working at Google: “you get to influence great products by working with

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\(^1\) Aghion et al. (2008) provides a rationale as to why such a separation can be optimal.
\(^2\) Arora et al (2017) show that the propensity of large firms to publish scientific results declined between 1980 and 2007, whereas their propensity to patent has increased and their patents continue to cite science in the same rate as before.
\(^3\) “From its inception, Genentech co-founder Herb Boyer, a pioneering gene splicer, insisted on publishing their discoveries in academic journals—as a stamp of quality—proving that they were in league with the best academic institutions, and to lure the best and brightest minds in bioscience to the company” (https://www.gene.com/stories/the-paper)
great engineers, while having the freedom to do research alongside some of the world experts in machine learning.⁴

A large literature analyses private investments in applied or proprietary research which can generate rents by providing a competitive advantage to innovating firms (e.g. Gilbert and Newbery 1982; Grossman and Shapiro 1987; Aghion et al 2005). Less is understood in relation to private investments in basic research whereby profit-seeking firms hire academically driven scientists who produce outputs (academic publications) that are not financially valuable and not subject to property rights. This paper proposes a theoretical rationalization for such investments.

Traditional contract theory models capture situations where agents (scientists) must be swayed to exert costly efforts so as to generate an output valuable to the principal (the firm). These cannot then explain a firm’s decision to allow any of its scientists devote time to non-profitable activities such as basic research as they. We rationalize private investments in basic research in the spirit of Murdock (2002) and Bénabou and Tirole (2003 and 2016) as we consider scientists who have an intrinsic motivation to pursue basic research. When agents have an intrinsic motivation to pursue a task, allowing them to allocate some time to this task is a valuable instrument that can control participation and incentives. Stern (2004) and Besley and Ghatak (2005) bring to light the financial benefits that arise from allowing agents devote time to their intrinsic, non-pecuniary, interests.

Specifically, we consider a monopolistic firm which faces a large community of scientists. Each is characterized by an intrinsic motivation to pursue basic research which is captured as the utility gathered from spending time on a personal academic agenda. We refer to this as the scientist’s type. This variable can reflect academic prestige measured by the number of publications and citations, in which case it could be contractible. Alternatively it can reflect a non-verifiable taste for science.

The contracted scientist may pursue basic research, which has no direct value for the firm, alongside a more applied agenda in line with the firm’s core business.⁵ We characterize the firm’s profit maximizing hiring decision and multi-task contract. The selection of a scientist with a stronger motivation captures a higher investment in basic research. The basic

⁴ https://research.google.com/workatgoogle.html
⁵ Rosenberg (1990) provides evidence that some firms explicitly encourage their employees to carry out curiosity driven research projects, alongside the more focused industrial projects dictated by top management.
framework assumes that all variables are contractible. It is then extended to cater first for moral hazard and then for adverse selection.

Two exogenous variables play an important role. The first is the opportunity cost associated with each scientist measured via a type-dependent reservation utility. We assume that a scientist with greater intrinsic motivation for basic research has potentially more alternative prospects and can therefore be subject to a greater opportunity cost. The second is the extent to which the research activities are complements or substitutes.

When all variables are contractible the firm targets the highest possible type when the opportunity cost is low. As the opportunity cost increases the firm reduces its investment. The contracted scientist is only allowed spend time on her agenda when research activities are complements or mild substitutes. The optimal investment decision balances two opposing forces. On the one hand, scientists with a greater academic inclination are potentially associated with a higher opportunity cost which puts upward pressure on the wage. On the other hand, allowing scientists to spend time on their agenda puts downward pressure on the wage as it increases the utility they get from accepting the contract.

Under moral hazard the firm must rely on an incentive payment (or bonus). A fixed wage, which is optimal when all variables are contractible, leads the scientist to put too much emphasis on her own agenda. In line with Holmstrom and Milgrom (1991) we show that this bias can be corrected by a proper allocation of property rights. Making the outcome of applied research valuable to the scientist turns her interest towards the firm’s agenda and enables the firm to restore the first best contract.

Under adverse selection the firm must write a contract that appeals only to the type it targets meaning that the contract must deter other scientists from considering the position. As a first result we show that the first best contract remains optimal when the opportunity cost is sufficiently low and the firm targets a scientist of the highest type (top academic). This is so because scientists face countervailing incentives. When a scientist applies for a contract aimed at a researcher with a higher intrinsic motivation, the value given to pursuing basic research is over-estimated and the wage lowered accordingly. When she applies for a contract aimed at a researcher with a lower intrinsic motivation, the value of her outside option is under-estimated. When the reservation utility is roughly the same for all, irrespective of their academic achievements, scientists have no incentive to inflate their motivation for basic
research as it only exaggerates the value given to pursuing their own agenda and the first best is incentive compatible.\(^6\)

As the opportunity cost increases the first best contract is no longer incentive compatible. To satisfy this constraint, the contract requires scientists to spend an amount of time on their own agenda that is positively correlated with their type. This imposes additional costs for the firm, which leads to lower investments in basic research. At the solution, the targeted type is required to spend too much time on basic research, more than what she would choose for herself. The perk ceases to be a privilege and becomes a constraint for both parties. The distortions triggered by the incentive constraint are more acute the higher are the opportunity costs and the degree of substitution between research activities. When research activities are substitutes, the firm ceases to invest in basic research as it hires a scientist with the lowest possible taste for science.

The structure of the paper is as follows. The next section presents a literature review. Section 3 describes the model. In section 4 we analyse the optimal contract when the types are verifiable and thus contractible. Within this section we cater for moral hazard. Section 5 deals with adverse selection. Section 6 concludes.

2. LITERATURE

The advantages, and potential disadvantages, of hiring top academics have been the focus of several empirical studies. See, for instance, Zucker et al. (2002), Simeth and Raffo (2013), Jong and Slavova (2014), Simeth and Cincera (2015), Arts and Veugelers (2016) for the benefits and Gittelman and Kogut (2003) for the costs.

Optimal contracting under multitasking has received much attention in the literature. Recently, some theoretical papers consider specifically the separation between applied and basic research. Lacetera and Zirulia (2012) characterize the optimal contracting of scientists by competing firms and show that the incentive payments depend on the interplay between the intensity of competition and the spillovers generated by the basic research outcomes. Following the Bayh-Dole Act, some raised concerns that academics would divert their attention to applied research and neglect the fundamental, basic research. Banal-Estañol and Macho-Stadler (2010) proves them wrong as they show that increased remuneration of commercial outcomes can actually lead researchers to select riskier projects.

\(^6\) In contract theory terms, one can argue that when the opportunity cost is low, the firm wants to hire the high type who is the “bad” type as scientists are tempted to dampen their enthusiasm for basic research.
Using data, Cockburn and Henderson (1998) show that incentivising in-house researchers to publish in academic journals allows to attract and retain top scientists which was corroborated by Herb Boyer, Genentech’s co-founder, who stated that “If we let them publish, they’ll come.” Stern (2004) provides empirical evidence that scientists are willing to accept lower wages when allowed to spend time on their agenda. Our analysis allows us to refine these findings as we show that, indeed, for given efforts, scientists pay to be scientists. But, with endogenous efforts the scientist’s wage is non-decreasing with her academic inclination. This brings us closer to Sauermann and Roach (2014), who find that “scientists who believe themselves to be of high ability and who train at top tier institutions have a higher price of publishing”.

Few theoretical papers analyze multitasking under adverse selection. Recently Bénabou and Tirole (2016) and De Fraja (2016) have addressed contracting in settings involving multitasking and adverse selection. Bénabou and Tirole (2016) bring to light the link between competition and the structure of compensation. They consider a setting where agents engage in two tasks, one of which leads to a contractible outcome positively correlated with the worker’s privately known type. They show that the distortions generated by adverse selection and the resulting incentive schemes depend on the market structure. Under monopsony the aim is to reduce informational rents. This is achieved via a contract that under-incentivizes the low type. As firms compete for workers their focus shifts to separating types. This is done thanks to high powered incentive schemes aimed at high types which deters them from engaging in tasks producing less contractible outcomes.

De Fraja (2016) characterizes the optimal public funding of research institutions by a government which favours applied research over basic research. Institutions view both types of research as perfect substitutes, they differ in their prestige and are privately informed about their ability to conduct research. The paper shows that, in the presence of adverse selection, the more efficient and more prestigious institutions are required to perform too much applied research so as to reduce their informational rents. In our setting, informational rents are not an issue. The firm is able to use the scientists’ countervailing incentives to eliminate rents. However, our the paper also brings to light the fact that, under adverse selection, a firm may be compelled to distort the scientists’ efforts and have her focus more intensively on one of the tasks. In De Fraja (2016) attention is biased towards the principal’s favoured task, namely

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7 https://www.gene.com/stories/the-paper
applied research. Here we show that incentive compatibility may oblige the firm to favour basic research.

3. THE MODEL

A firm wishes to contract a scientist and faces a large pool of heterogeneous candidates. Each is characterized by her intrinsic motivation to pursue basic research captured by the parameter $\phi \in [0, \bar{\phi}]$ which constitutes her type.

When hired, the scientist is offered a contract specifying the remuneration ($w$) and, when contractible, the efforts she must dedicate to pursuing her personal academic research agenda ($e_p$) and the firm’s research agenda ($e_F$).

Subject to dedicating time to her own agenda ($e_p > 0$), she produces an output of an academic nature that has direct benefits for her only. Subject to spending time on the firm’s agenda ($e_F > 0$) she produces an output of a commercial or industrial nature that benefits the firm (and possibly the scientist through a bonus payment).

The firm’s overall profits are subject to uncertainty and given by

$$\Pi = e_F \pi - \beta w. \quad (1)$$

The variable $\beta \geq 1$ accounts for potential additional shadow costs (such as taxes or costs of raising capital). Finally, $\pi$ is a random variable reflecting the marginal value of the effort $e_F$. It is given by

$$\pi = \tau + \rho \phi + \bar{\epsilon}, \quad (2)$$

where $\tau$ measures the marginal productivity of labour and $\rho$ measures the productivity effect (as per the terminology in Stern (2004)). The term $\bar{\epsilon}$ is a random variable with zero mean. Consequently the expected values of $\pi$ and $\Pi$ are given by

$$\pi(\phi) = \tau + \rho \phi \quad \text{and} \quad \Pi(\phi) = e_F \pi(\phi) - \beta w. \quad (3)$$

We consider that scientists are risk neutral with respect to income. The objective function of a scientist of type $\phi$ is given by

$$U(\phi; e_F, e_p) = w + e_p \phi - C(e_F, e_p), \quad (4)$$

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8 Bénabou and Tirole (2016) consider a model where the type and the effort are substitutes whereby the returns to the firm are ($e_F + \phi$). We consider instead a situation where the firm can only gather $\pi(\phi)$ provided $e_F > 0$. 

Where \( C(e_F, e_P) \) is the cost of exerting the efforts:
\[
C(e_F, e_P) = \frac{1}{2}(e_F^2 + e_P^2) - \gamma e_F e_P. \tag{5}
\]
As indicated in (4), the type measures the marginal utility associated with \( e_p \). It is interpreted as scientist’s intrinsic motivation to do basic research.

The parameter \( \gamma \) measures the extent to which research activities may be complementary \((\gamma > 0)\), substitutes \((\gamma < 0)\) or independent \((\gamma = 0)\). Concretely, the sign and size of this variable is likely to be field dependent.

The following assumption guarantees that the optimal hiring strategy of the firm is non-negative.

**Assumption:** We assume that \( \rho + \beta \gamma \geq 0 \).

This requirement is quite intuitive. Basically if scientists become less productive as their academic inclination (or type) increases \((\rho < 0)\) and if the research activities were substitutes \((\gamma < 0)\), then the firm’s optimal type would be \( \phi = 0 \). \(^{10}\)

This requirement implies the firm hires a scientist with a positive taste for science provided the productivity effect is positive when research activities are substitutes. But, when research activities are complementary, an interior solution can emerge even when the productivity parameter is (slightly) negative.

When she refuses the contract, the scientist has outside options which grants her a reservation utility \( U(\phi) \). Specifically, we assume that the reservation utility is expressed as
\[
U(\phi) = w + \frac{\sigma}{2} \phi^2, \tag{6}
\]
where \( w > 0 \) is the reservation wage and \( \sigma \geq 0 \) captures the fact that scientists with a higher academic motivation may have access to more and better outside options, specifically in research oriented institutions.

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\(^9\) The wage could depend on the realization of the random variable \( \pi \) in which case the scientist would take its expectation.

\(^{10}\) Arts and Veugelers (2016) supports the fact that academically driven scientists are more productive so that \( \rho > 0 \).
4. OPTIMAL CONTRACTS WHEN THE TYPE IS VERIFIABLE.

4.1 SYMMETRIC INFORMATION

In this section we assume that each firm can perfectly verify the scientists’ types as well as the effort levels. Therefore the firm maximizes its expected profits:

$$\max_{w,\phi,e_P,e_F} \Pi(\phi),$$

where $\Pi(\phi)$ is given by (3) subject to a participation constraint (for each type):

$$U(\phi; e_P, e_F) \geq U(\phi)$$

where $U(.)$ is given by (6), and a feasibility constraint

$$0 \leq \phi \leq \bar{\phi},$$

$$e_P \geq 0.$$  \hfill (9)

The objective function is maximized when the participation constraint binds. Thus (7) can be rewritten as

$$\max_{\phi,e_P,e_F} e_F \pi(\phi) + \beta e_P \phi - \beta \left[ w + \frac{\sigma}{2} \phi^2 + C(e_F, e_P) \right].$$

Both efforts are associated with marginal benefits: $\pi(\phi)$ for $e_F$ and $\beta \phi$ for $e_P$. The latter captures the preference effect highlighted in Stern (2004): allowing scientists to spend time on their agenda exerts a downward pressure on the wage. Setting $e_P > 0$ is an alternative form of compensation, a perk, which facilitates participation. If, in addition, research activities are complements, then the firm gets further benefits from setting $e_P > 0$ as shown below.

Proposition 1: Optimal contract when research activities are complements or independent ($\gamma \geq 0$).

When $\gamma \in [0,1]$ the optimal hiring decision is such that

$$\phi^* = \begin{cases} \bar{\phi} & \text{when } \sigma \leq T^*, \\ \frac{T(\rho + \gamma \beta)}{\delta^*} & \text{when } \sigma \geq T^*, \end{cases}$$

where $\delta^* = \beta^2 \sigma (1 - \gamma^2) - (\beta^2 + \rho^2 + 2 \gamma \beta \rho)$ and

\footnote{Adding the constraint according to which we must have $e_F \geq 0$ is irrelevant as the analysis will show. The firm gets no revenue unless this constraint applies. Therefore all solutions will be such that $e_F > 0$.}
\[ T^* = 1 + \frac{(\rho + \beta \gamma)(\tau + \phi(\rho + \beta \gamma))}{\beta^2 \phi (1 - \gamma^2)} \]

The optimal effort levels are given by

\[ e_F^* = \frac{\pi(\phi^*) + \gamma \beta \phi^*}{\beta (1 - \gamma^2)} \quad \text{and} \quad e_P^* = \frac{\beta \phi^* + \gamma \pi(\phi^*)}{\beta (1 - \gamma^2)}. \]

**Proof:** See Appendix.\[\]

The value for \( T^* \) is calculated such that \( \frac{\tau (\rho + \gamma \beta)}{\delta^*} \leq \phi \Leftrightarrow \sigma \geq T^* \). Hence, the solution is continuous as depicted in the figure below.

![Figure 1: Optimal hiring decision as a function of the opportunity cost.](image)

The opportunity cost (\( \sigma \)) constitutes an important barrier to hiring top scientists. The productivity of labour (\( \tau \)) has a positive impact on the hiring decision. Note that when \( \tau = 0 \) the hiring decision becomes discontinuous and we have

\[ \phi^* = \begin{cases} \phi & \text{when } \sigma \leq T^*, \\ 0 & \text{when } \sigma \geq T^*. \end{cases} \]

An increase in taxes or the cost of raising capital (measured by the parameter \( \beta \)) is detrimental for two reasons. First, \( \phi^* \) is non-increasing in \( \beta \). Second, as \( \beta \) increases the
parameter $T^*$ decreases meaning that the firm selects the highest type for fewer values of the parameter $\sigma$. Finally, the firm selects the highest type for a greater range of parameters when research activities exhibit a greater complementarity (that is when $\gamma \to 1$).

We now turn our attention to the optimal effort levels. In the Appendix we show that the firm’s expected profits are maximized at

$$e_F = \frac{\pi(\phi) + \gamma \beta \phi}{\beta(1 - \gamma^2)}$$

and

$$e_P = \frac{\beta \phi + \gamma \pi(\phi)}{\beta(1 - \gamma^2)}$$

for any $\phi$. (12)

When research activities are independent or complements so that $0 \leq \gamma \leq 1$, both of these values are positive and form a solution. Thus, in sectors where basic and applied research activities are complements (or independent) the firms always encourage the scientist to pursue both activities. Higher types must dedicate greater efforts to both activities and since we have $\frac{\partial e_F}{\partial \phi} > \frac{\partial e_P}{\partial \phi}$ higher types devote relatively more time to the firm’s agenda.

An increase in the marginal productivity of labour ($\tau$) or in the productivity effect ($\rho$) increases $e_F$. An increase in the shadow cost $\beta$ leads to lower values of $e_F$. None of these exogenous variables have an impact on $e_P$ when $\gamma = 0$. When $\gamma > 0$, they impact $e_P$ as they do $e_F$ but to a lesser extent: $|\frac{\partial e_P}{\partial x}| < |\frac{\partial e_F}{\partial x}|$ for $x \in \{\tau, \rho, \beta\}$.

Let us now consider research activities that are substitutes. In this case, research activities become strategic substitutes (see Appendix). Hence, allowing the scientist to work on her own agenda is potentially detrimental to the firm despite the presence of a preference effect since these variables become strategic substitutes.

**Proposition 2: Optimal contract when research activities are substitutes.**

When research activities are mild substitutes, that is when $\gamma \in \left[-\frac{\beta \phi}{\tau + \rho \phi}, 0\right]$, the optimal contract is as follows:

- For $\sigma < -\frac{\rho}{\beta \gamma}$ the optimal hiring decision remains $\phi^*$ as defined in Proposition 1 and the effort levels are given by $e_F^*$ and $e_P^*$.
- For $\sigma \geq -\frac{\rho}{\beta \gamma}$ the optimal hiring decision is $\phi^{**} = \frac{\tau \rho}{\beta^2 \sigma - \rho^2}$ and the effort levels are given by $e_F = \frac{\tau + \rho \phi^{**}}{\beta}$ and $e_P = 0$. 


When research activities are strong substitutes, that is when \( \gamma \in \left[ -1, -\frac{\beta \delta}{\tau + \rho \phi} \right] \), the firm never encourages the scientist to pursue her own agenda and sets \( e_p = 0 \). The optimal hiring decision is given by

\[
\phi^{**} = \begin{cases} 
\phi & \forall \sigma \leq T^{**}, \\
\frac{\tau \rho}{\beta^2 \sigma^2 - \rho^2} & \forall \sigma \geq T^{**}, 
\end{cases}
\]

where \( T^{**} = \frac{\tau \rho + \rho^2 \phi}{\rho^2 \phi} \).

**Proof:** See Appendix 1.

Figure 2 below gives a visual representation of the optimal contract for all \(-1 \leq \gamma \leq 1\).
When research activities are complements, we know that the firm always sets $e_p > 0$. It targets the highest type when the opportunity cost is sufficiently low ($\sigma \leq T^*$). When research activities are mild substitutes these two possibilities (the fully interior solution or the one where the top scientist is targeted) remain optimal. However, when research activities become stronger substitutes it is no longer optimal to allow the scientist to pursue her own agenda. In this case the optimal constrained candidate is such that $e_F = \frac{\pi(\phi)}{\beta}$ and $e_p = 0$ and the associated hiring strategy is then given by $\phi^{**}$ where the value for $T^{**}$ is such that

$$\frac{\tau \rho}{\beta^2 \sigma - \rho^2} \leq \phi \iff \sigma \geq T^{**}.$$  
(13)

Notice that the optimal contract is continuous in both $\gamma$ and $\sigma$.

The firm only allows the scientists to pursue their agenda when research activities are complements or independent or else when the degree of substitution is low. In this last case it only allows the scientists with higher types to pursue their agenda.

Before we extend the analysis and allow for moral hazard, we examine the correlation between the wage and the scientist’s ability. We pay particular attention to the result highlighted in Stern (2004) and Arts and Veugelers (2016) according to which scientists pay to be scientists, meaning that scientists who are allowed to pursue their own agenda receive lower wages.

**Lemma 1:** The wage is increasing in the ability provided the productivity parameter $\rho$ is sufficiently large even when $\sigma = 0$.

**Proof:** It is trivial to prove that the wage is increasing in $\phi$ when the scientist is not allowed to dedicate effort to her own agenda. We therefore focus on the case where $e_p > 0$. Since the participation constraint binds we have

$$w^* = w + C(e_F, e_p) - e_p \phi + \frac{\sigma}{2} \phi^2,$$  
(14)

where $C(e_F, e_p)$ is given by (5). Subbing in the optimal efforts, we have

$$w^* = w + \frac{1}{2 \beta^2 (1 - \gamma^2)} \left[ \pi^2 - \beta^2 \phi^2 \right] + \frac{\sigma}{2} \phi^2.$$  
(15)

Clearly $\sigma > 0$ contributes to a positive correlation between the wage and the type. But, even when $\sigma = 0$ we have
\[
\frac{dw^*}{d\phi} = \frac{1}{\beta^2 (1-\gamma^2)} \left( \rho \tau + \phi (\rho^2 - \beta^2) \right). 
\]

(16)

The above is positive when \( \rho \) is large enough. □

Our findings allow us to refine those presented in Stern (2004). For any given efforts (or any given work agenda) the wage is non-increasing with the scientist’s ability when \( \sigma = 0 \) since \( \frac{dw}{d\phi} = -e_P \). However, if we factor in the endogeneity of the efforts, the wage can be increasing in \( \phi \). If one interprets efforts as a level of responsibility given to a scientist, our results suggest that scientists with greater ability have more responsibilities and are therefore paid more to compensate them.

3.2 MORAL HAZARD.

The purpose of this section is to show that moral hazard is not an issue provided the firm can introduce a bonus pay. While this is a well-known result, we also show that the optimal bonus depends solely on the value of \( \beta \).

Assume that the firm can only verify the realization of the random variable \( \tilde{\chi} = e_F (\tau + \rho \phi + \tilde{\epsilon}) \). It can no longer verify the effort level. Assume furthermore that the firm pays a fixed wage \( w \). Given the wage, that type \( \phi \) scientist solves

\[
\max_{e_P, e_F} \bar{w} + e_P \phi - C(e_F, e_P),
\]

s.t. \( e_F \geq 0 \) and \( e_P \geq 0 \).

If research activities are complements the scientist selects \( e_P = \frac{\phi}{(1-\gamma^2)} \) and \( e_F = \frac{\gamma \phi}{(1-\gamma^2)} \) and gives greater emphasis to her own agenda. When the research activities are substitutes or independent she selects \( e_P = \phi \) and \( e_F = 0 \). Whether activities are complements or substitutes, the efforts chosen by the scientist are sub-optimal decisions for the firm.

Assume that the firm sets a linear wage to address the incentive issue highlighted above. Thus let \( w = t + s \chi \) where \( t \) is a fixed fee, \( \chi \) denotes a realization of the random variable \( \tilde{\chi} \) and \( s \) denotes a share of the profits. We have \( 0 < s \leq 1 \). ¹²

**Lemma 2:** Given a wage \( w = t + s \chi \) the following occurs:

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¹² We know that setting \( s = 0 \) cannot be optimal.
• When research activities are complements or independent, all types exert positive efforts and we have
\[ e_F = \frac{s\pi(\phi) + \gamma \phi}{1 - \gamma^2} \text{ and } e_P = \frac{\phi + \gamma s\pi(\phi)}{1 - \gamma^2}. \]

• When research activities are mild substitutes and \( \gamma \in \left[ -\frac{\phi}{s(\tau + \rho \phi)}, 0 \right] \), only types \( \phi \geq \frac{-sy\tau}{1 + sy\rho} \) exert positive efforts given above while lower types dedicate \( e_F = s\pi(\phi) \) to the firm’s agenda and \( e_P = 0 \).

• When research activities are strong substitutes and \( \gamma \in \left[ -1, -\frac{\phi}{s(\tau + \rho \phi)} \right] \), all types dedicate \( e_F = s\pi(\phi) \) to the firm’s agenda and \( e_P = 0 \).

Proof: The proof is straightforward. For any \( \gamma > 0 \) the Hessian matrix is negative definite provided \((1 - \gamma^2) > 0\) which holds for all \(-1 \leq \gamma < 1\). This strict concavity of the objective function means that there is a unique solution which is possibly constrained. □

A greater bonus rate, \( s \), has a direct positive impact on \( e_F \). It also broadens the interval \( \left[ -1, -\frac{\phi}{s(\tau + \rho \phi)} \right] \) for which we systematically have \( e_P = 0 \). Said differently, when the bonus rate increases the scientist withdraws from her own agenda for a wider range of degrees of substitution between activities.

Taking these efforts as given, the firm decides upon \( t, s \) and \( \phi \) so as to maximize its expected profits subject to constraints (8) and (9).

It is optimal to set the fixed fee \((t)\) such that the participation constraint (8) binds. The firm then solves
\[ \max_{\phi, s} e_F\pi(\phi) - \beta \left[ w + \frac{\sigma}{2} \phi^2 - e_P \phi + C(e_F, e_P) \right], \]
subject to (9) where \( e_F \) and \( e_P \) are characterized in Lemma 2.

Proposition 3: Optimal contract under moral hazard

Setting \( s = \frac{1}{\beta} \) allows the firm to implement the first best contract and adopt the same hiring strategies as the ones depicted in Propositions 1 and 2.

Proof: See Appendix.
The optimal share of the profits given to the scientist depends solely on $\beta$. When $\beta$ increases, the firm reduces the incentive payment: a larger portion of the wage is fixed and uncorrelated to the profits because the firm seeks to implement lower efforts.

The conclusion from this short subsection is that moral hazard has very little impact if any when the firm faces no restrictions in relation to the bonus rate and fixed fee. If $\beta = 1$ the firm must pledge all of the revenue to the scientist. It may then have to set a negative fixed fee such that the scientist’s utility matches her reservation utility. Setting up a negative fixed fee may not be a possibility in which case the bonus rate would be bounded above and incentives could no longer be aligned.

5. OPTIMAL CONTRACTS UNDER ADVERSE SELECTION

So far we considered that the scientist’s ability was verifiable. This can be motivated by the fact that the quality of academic achievements, such as the number of publications or citations, is verifiable and may serve as a proxy. However, if we consider that $\phi$ is more representative of a taste for science, this variable is not necessarily observable and certainly more difficult to verify. In this section we assume that it is not contractible. We do not remove the assumption that there are many scientists of a given scientific ability meaning that the firms can issue a single contract aimed at a specific type.

Let us consider the problem of a firm who wants to contract a scientist of type $\hat{\phi}$. It issues a contract which specifies the wage $\hat{w}$, and the effort levels that it wants the scientist to exert $e_k(\hat{\phi})$ with $k = P, F$.\(^\text{13}\)

The scientist of type $\hat{\phi}$ accepts the contract provided

$$\hat{w} + e_P(\hat{\phi}) (\phi) - C(\hat{\phi}) \geq U(\hat{\phi}),$$

(17)

where $C(\hat{\phi}) = \left[ \frac{1}{2} \left( (e_F(\hat{\phi}))^2 + (e_P(\hat{\phi}))^2 \right) - \gamma e_F(\hat{\phi}) e_P(\hat{\phi}) \right]$.\(^\text{13}\)

To make sure that scientists with a different type do not apply for the same contract, we must have:

$$\hat{w} + e_P(\hat{\phi}) (\phi') - C(\hat{\phi}) \leq U(\phi'),$$

(18)

\(^\text{13}\) An equivalent approach consists in considering that the firm offers a menu of contracts stipulating the wage and effort levels as well as a probability of selecting each possible type. In our approach we characterize the contract that will appeal only to the optimally selected type.
for any $\phi' \neq \phi$. Notice that the cost $C(\phi)$ is the same as above because it only depends on the level of efforts to be exerted.

**Proposition 4: Necessary and sufficient conditions for incentive compatibility.**

A contract targeting a top scientist $\{\overline{e}_F, \overline{e}_P, \overline{w}\}$ satisfies incentive compatibility and leaves no rents to the top scientist if and only if

$$\overline{e}_P \geq \sigma \overline{\phi} \quad \text{and} \quad \overline{w} = \underline{w} - \overline{e}_P \overline{\phi} + C(\overline{e}_F, \overline{e}_P) + \frac{\sigma}{2} \overline{\phi}^2.$$

A contract targeting a scientist with specific ability $\hat{\phi} < \overline{\phi}$ $\{\hat{e}_F, \hat{e}_P, \hat{w}\}$ satisfies incentive compatibility and leaves no rents to the targeted scientist if and only if

$$\hat{e}_P = \sigma \hat{\phi} \quad \text{and} \quad \hat{w} = \underline{w} - \hat{e}_P \hat{\phi} + C(\hat{e}_F, \hat{e}_P) + \frac{\sigma}{2} \hat{\phi}^2.$$

The more academically oriented the targeted scientist is the greater the effort the firm has to allow her to dedicate to her own research agenda.

**Proof:** We focus here on the case where the firm targets the highest type. The remaining of the proposition is demonstrated in Appendix 4.

It is very clear that the participation constraint binds if and only if $\overline{w} = \underline{w} - \overline{e}_P \overline{\phi} + C(\overline{e}_F, \overline{e}_P) + \frac{\sigma}{2} \overline{\phi}^2$. Let us focus on incentive compatibility. Consider a scientist of type $\phi' < \overline{\phi}$. If she is offered the contract $\{\overline{e}_P, \overline{e}_P, \overline{w}\}$ aimed at type $\overline{\phi}$ she gets

$$U(\phi'; \overline{w}, \overline{e}_P, \overline{e}_F) = \overline{w} + \overline{e}_P \phi' - C(\overline{\phi}) + \frac{\sigma}{2} \overline{\phi}^2 = \underline{w} + \frac{\sigma}{2} \overline{\phi}^2 - \overline{e}_P \phi'. \quad (19)$$

The above is a linear function of $\phi'$ of constant positive slope $\overline{e}_P$.

If she does not apply for this contract she gets her reservation utility. Note that $\underline{U}(\phi) = \underline{w} + \frac{\sigma}{2} (\phi')^2$ is a convex function of $\phi'$. Notice of course that at $\phi' = \overline{\phi}$ the utility and the reservation utility are equal. Figure 3 below shows why the contract $\{\overline{e}_F, \overline{e}_P, \overline{w}\}$ attracts the highest type if and only if the slope of the linear utility is at least equal to the slope of the reservation utility at $\overline{\phi}$. 


Clearly, when $\overline{e}_p > \sigma \overline{\phi}$, none of the types other than the highest type want to accept the contract $\{\overline{e}_F, \overline{e}_P, \overline{w}\}$ as their reservation utility is above their utility from accepting the contract. This is not the case when $\overline{e}_p < \sigma \overline{\phi}$ whereby some types below $\overline{\phi}$ would also be better-off accepting the contract. □

Proposition 4 shows that the allocation of tasks plays a crucial role under adverse selection by enabling the separation of types. Notice that the proposition holds irrespectively of the value of $\gamma$. However, the impact the asymmetry of information has does depend on whether activities are substitutes or complements. Since incentive compatibility requires that the scientist be allowed spend some time on her agenda the presence of adverse selection has a greater cost when research activities are substitutes and the first best contract imposes $e_p = 0$. But what is maybe more interesting is that when top scientists are targeted, some of the first best contracts may actually satisfy incentive compatibility.
Proposition 5: The symmetric information contract is incentive compatible when research activities are complements, the firm targets a top scientist and $\sigma \leq T$ where

$$\hat{T} = 1 + \frac{\tau \gamma + \gamma \bar{\phi} (\gamma \beta + \rho)}{\beta \bar{\phi} (1 - \gamma^2)}.$$ 

Proof: One can easily show that $\hat{T}$ is such that $e_P \geq \sigma \bar{\phi} \Leftrightarrow \sigma \leq \hat{T}$. Finally one can easily show that $\hat{T} < T^*$ meaning that the regions for which the highest type is optimal and for which the effort $e_P \geq \sigma \bar{\phi}$ overlap.

The reason why the first best contract can remain optimal is because scientists face countervailing incentives. Under countervailing incentives the "bad" type is endogenously defined.\textsuperscript{14}

On the one hand a scientist’s intrinsic motivation for basic research puts downward pressure on the wage she is willing to accept when allowed to pursue her agenda. But, on the other hand, a higher reservation utility puts upward pressure on the wage. When $\sigma$ is low enough, and for given efforts, scientists with a low ability are not tempted to accept a contract aimed at a scientist with high ability because it over-estimates their enthusiasm for personal research. In such cases the firm is seeking to hire the highest type who may coincide with the "bad" type. When $\sigma$ is high, and for given efforts, scientists with a low ability are tempted to accept a contract aimed at a scientist with high ability because it over-estimates their outside options.

A question that remains is whether some of the first best contracts targeting types below $\bar{\phi}$ can also remain optimal. Corollary 1 provides the answer.

Corollary 1: Among the first best contracts none of those targeting a type $\phi^* < \bar{\phi}$ are incentive compatible.

Proof: Incentive compatibility requires that $e_{P^*} = \sigma \phi^*$. This holds provided $\sigma = 1$. However, according to Proposition 1, $\phi^* < \bar{\phi} \Leftrightarrow \sigma > T^* > 1$. Hence whenever $\phi^* < \bar{\phi}$ we have $e_{P^*} < \sigma \phi^*$ which violates incentive compatibility.

\textsuperscript{14} The bad type is the one that has no incentive to misrepresent his type when the first best is implemented.
**Optimal contract when the first best is not incentive compatible, and research activities are independent or complements.**

To characterize the optimal hiring decision, the firm maximizes (11) subject to the constraint \( e_p(\hat{\phi}) = \sigma \hat{\phi} \) since the participation constraint can hold with equality. As \( \sigma \) increases the firm must allow the contracted scientist to spend an increasing amount of time on her agenda and on the firm’s agenda when \( \gamma \in [0,1] \). This puts pressure on the wage as the cost associated with the effort levels increases. To curb this increase in the cost, the firm distorts its hiring decision from the first best and targets a lower type.

**Proposition 6: Optimal contract under adverse selection when \( \sigma > \hat{T} \) and \( \gamma \in [0,1] \).**

When \( \gamma \in [0,1] \) the optimal hiring decision is such that

\[
\phi^{***} = \begin{cases} 
\frac{\bar{\phi}}{\tau(\rho + \gamma \beta \sigma)} & \text{when } \sigma \leq T^{***}, \\
\frac{\tau(\rho + \gamma \beta \sigma)}{\delta^{***}} & \text{when } \sigma \geq T^{***},
\end{cases}
\]

where \( \delta^{***} = \beta^2 \sigma (\sigma - 1) - (\rho + \gamma \beta \sigma)^2 \) and \( T^{***} \) is such that

\[
\frac{\tau(\rho + \gamma \beta \sigma)}{\delta^{***}} \leq \bar{\phi} \iff \sigma \geq T^{***}.^{15}
\]

The optimal effort levels are given by

\[
e^{***}_f = \frac{1}{\beta} [\tau + \phi^{***}(\rho + \gamma \beta \sigma)] \quad \text{and} \quad e^{***}_p = \sigma \phi^{***}.
\]

**Proof:** See Appendix.

An important result of this paper is stated in the corollary below. Under symmetric information the main deterrent to hiring top academics is the opportunity cost associated with their outside option. Under adverse selection the opportunity cost interferes through a different channel.

**Corollary 2:** Under adverse selection, any interior solution is supported by the fact that incentive compatibility requires that the firms set \( e_p \) beyond what the scientist would choose for herself.

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^{15} See the Appendix for an expression of \( T^{***} \).
Proof: The first order condition can be written as
\[
\frac{\partial \Pi}{\partial \phi} = e_F \rho + \beta [e_p - \sigma \phi] + \beta \frac{\partial U}{\partial e_p} \frac{de_p(\phi)}{d\phi} = 0. \tag{20}
\]
Given that the second term vanishes under incentive compatibility, we can only have an interior solution provided the last term is non-positive so that
\[
\frac{\partial U}{\partial e_p} \bigg|_{e_p=\sigma \phi} < 0. \tag{21}
\]
The above suggests that the firm requests that scientists spend *too much* time on their own research agenda. \(\Box\)

In other words, the presence of adverse selection leads the firm to put too much emphasis on the time spent on the personal agenda. When research activities are complements this also means that it must increase the overall workload. Thus, adverse selection leads to under-investment because the firm must implement high levels of efforts to guarantee that the contract they issue is incentive compatible.

- **Optimal contract when the first best is not incentive compatible: allowing for substitutes.**

When activities are substitutes the firm decreases \(e_F\) as \(e_p\) rises. This lowers the firm’s returns but it also puts less pressure on the wage. Overall though the impact of adverse selection is more severe when research activities are substitutes because the firm is less inclined to allow the scientist to spend time on her agenda. Setting \(e_p = 0\) is no longer an option unless of course the firm targets the lowest type.

In the Appendix we show that the partial derivative of the expected profits with respect to the type is given by
\[
\frac{\partial \Pi}{\partial \phi} = \frac{1}{\beta} [\tau (\rho + \gamma \beta \sigma) - \delta^{***} \phi],
\]
where \(\delta^{***} = \beta^2 \sigma (\sigma - 1) - (\rho + \gamma \beta \sigma)^2\).

When \(\gamma \geq 0\) the marginal benefit associated with type \(\phi = 0\) is non-negative (it is nil only when the marginal productivity of a worker \(\tau = 0\)). This means that the firm has incentives to hire a scientist with a positive type. When, in addition, the opportunity cost is low and we
have $\delta^{***} < 0$, the expected profits increase with the type and it is optimal to select the highest type. As a matter of fact it is optimal to target the highest type for positive, low values of $\delta^{***}$ so that the solution is continuous as shown in proposition 6.

When $\gamma < 0$, the sign of $\rho + \gamma \beta \sigma$ is no longer systematically positive meaning that the marginal benefit associated with type $\phi = 0$ could be negative. One can easily show that $(\rho + \gamma \beta \sigma) > 0 \Rightarrow \delta^{***} > 0$. This means that when the lowest type is associated with a negative marginal productivity, so are all other types. In this case the expected profits are decreasing with the type so the more academically inclined scientists are worth less and less to the firm because they must be allowed to spend too much time on their personal agenda. Hence, the firm selects the lowest type. But this solution does not prevail for all values of $\sigma$ and $\gamma$ as shown in the graph below.

![Figure 4: The optimal hiring decision under adverse selection for all possible combinations of $\sigma$ and $\gamma$.](image)

Clearly adverse selection acts as a deterrent when research activities are substitutes because the firm must select a low type if it wants to reduce effort dedicated to the personal agenda.
6. CONCLUSION

Hiring scientists and allowing them to wonder off and follow their own research agenda can nurture innovations. Google’s 20% rule allows employees to spend time on their side projects one day a week and innovations like Gmail and AdSense were originated thanks to such policies.\footnote{https://www.fastcompany.com/3015963/google-took-its-20-back-but-other-companies-are-making-employee-side-projects-work-for-them; http://www.forbes.com/sites/johnkotter/2013/08/21/googles-best-new-innovation-rules-around-20-time/#5ff3a50268b8} It is less trivial to understand why a profit maximizing firm would allow scientists to indulge in research of a more academic nature, which is not motivated by the invention of a new product or technology.

This paper shows that when scientists have an intrinsic motivation to pursue basic research, allowing them to dedicate some of their time to their own agenda is a valuable instrument that can control participation and incentives.

We show that the decision to hire academically oriented scientists is profit maximizing in a context where basic and applied research activities are not highly substitutable and when the opportunity cost associated with deterring scientists from remaining in academia is not too large. This result remains robust to the introduction of moral hazard.

Under adverse selection the allocation of tasks, and in particular the effort dedicated to basic research is used to address incentive issues. The more academically inclined the targeted scientist is, the more effort she must dedicate to her own agenda. When the opportunity cost is low the first best contract remains optimal. But, when the opportunity cost is large enough, this constraint pushes the firm to under-invest and select less academically inclined scientists. Under-investment is not driven by the need to control informational rents. Instead it addresses the increase in costs that results from requiring that the scientist spend an excessive amount of time on her own agenda.
APPENDIX

Appendix 1: Proof of Proposition 1 and 2.

In what follows we ignore the constraint $e_F \geq 0$ because, as we show below, this constraint is always satisfied.

The first order conditions to the maximization problem given by equations (7) to (10) are

\[
\frac{\partial \Pi}{\partial e_F} = \pi(\phi) - \beta(e_F - \gamma e_P) = 0 \\
\frac{\partial \Pi}{\partial e_P} = \beta(e_P + \gamma e_F) - \lambda_1 = 0, \\
\frac{\partial \Pi}{\partial \phi} = \beta e_P + \rho e_F - \beta \sigma \phi - \lambda_2 = 0, \\
\lambda_1 e_P = 0 \text{ and } \lambda_2(\phi - \bar{\phi}) = 0.
\]

The variables $\lambda_1$ and $\lambda_2$ are the Lagrange multiplier measuring the shadow costs of the constraints $e_P \geq 0$ and $(\phi - \bar{\phi}) \geq 0$ respectively.

The objective function is strictly concave in $(e_F, e_P)$ since the Hessian matrix associated to these variables is negative definite provided $(1 - \gamma^2) > 0$ which holds for all $-1 < \gamma < 1$.

This means that, for any given type, setting

\[
e_F = \frac{\pi(\phi) + \gamma \beta \phi}{\beta(1 - \gamma^2)} \text{ and } e_P = \frac{\beta \phi + \gamma \pi(\phi)}{\beta(1 - \gamma^2)}
\]

is the unique unconstrained candidate for a maximum. Notice the following:

- By assumption $e_F \geq 0$ for any $\gamma \in [-1, 1]$.
- For any $\gamma \in [0,1]$ $e_P \geq 0$ as well meaning that for such values of $\gamma$ the values above are optimal.

When research activities are complements or independent.

The optimal effort levels are given above and we have (after simplifications)

\[
\frac{\partial \Pi}{\partial \phi} = \frac{1}{\beta(1 - \gamma^2)}[\pi(\gamma \phi + \rho) - \delta^* \phi - \lambda_2, 23]
\]

where $\delta^* = \beta^2 \sigma(1 - \gamma^2) - (\beta^2 + \rho^2 + 2\gamma \beta \rho)$.  

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When $\delta^* \leq 0$ the objective function is increasing with the type. Therefore $\lambda_2 > 0$ and it is optimal to set $\phi^* = \bar{\phi}$.

When $\delta^* > 0$ there exists a potential interior solution $\phi^* = \frac{\tau (\rho + \gamma \beta)}{\delta^*}$. And we have $\frac{\tau (\rho + \gamma \beta)}{\delta^*} \leq \bar{\phi} \iff \sigma \geq T^*$. Notice that at $\sigma$ such that $\delta^* = 0$ setting $\phi^* = \bar{\phi}$ is optimal. Therefore, we must have $\delta^* > 0$ to reach an interior solution and the only threshold that matters is $T^*$. Thus,

- For any $\sigma \in [0, T^*]$ it is optimal to select $\phi^* = \bar{\phi}$ either because the objective function increases with the type or because $\frac{\tau (\rho + \gamma \beta)}{\delta^*} \geq \bar{\phi}$.
- For any $\sigma \geq T^*$ it is optimal to select $\phi^* = \frac{\tau (\rho + \gamma \beta)}{\delta^*}$.

When research activities are substitutes.

The solution characterized previously remains optimal when $e^*_p > 0$.

When the exogenous parameters are such that $e^*_p < 0$ we must consider a constrained solution whereby $e_p = 0$. In this case it follows from the first order conditions that $e_F = \frac{\pi(\phi)}{\beta} \geq 0$. Given these effort levels we have (after simplifications)

$$\frac{\partial \Pi}{\partial \phi} = \frac{1}{\beta} \left[ \tau \rho - (\beta^2 \sigma - \rho^2) \phi \right] - \lambda_2. \tag{24}$$

When $(\beta^2 \sigma - \rho^2) \leq 0$, the objective function is increasing with the type. Therefore $\lambda_2 > 0$ and it is optimal to set $\phi^* = \bar{\phi}$. When $(\beta^2 \sigma - \rho^2) > 0$ there exists a potential interior solution $\phi^{**} = \frac{\tau \rho}{(\beta^2 \sigma - \rho^2)}$. And we have $\frac{\tau \rho}{(\beta^2 \sigma - \rho^2)} \leq \bar{\phi} \iff \sigma \geq T^{**}$. For the same reasons as before the following applies:

- For any $\sigma \in [0, T^{**}]$ it is optimal to select $\phi^* = \bar{\phi}$ either because the objective function increases with the type or because $\frac{\tau \rho}{(\beta^2 \sigma - \rho^2)} \geq \bar{\phi}$.
- For any $\sigma \geq T^{**}$ it is optimal to select $\phi^{**} = \frac{\tau \rho}{(\beta^2 \sigma - \rho^2)}$.

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17 See the expression for $T^*$ in proposition 1.
18 See the expression for $T^{**}$ in proposition 2.
In figure 2 we represent the functions $\sigma = T^*$ and $\sigma = -\frac{\rho}{\beta} \Leftrightarrow e^*_p = 0$. It is straightforward to prove that these intercept at $\gamma = -\frac{\beta \phi}{\tau + \rho \phi} \in ]-1,0[$ at which point $\sigma = T^{**}$.

**Appendix 2: Proof of Proposition 3:**

The partial derivatives with respect to all variables are given by

$$\frac{\partial \Pi}{\partial s} = \frac{(\pi(\phi))^2}{1 - \gamma^2} (1 - s\beta),$$

$$\frac{\partial \Pi}{\partial \phi} = \beta e_F + \rho e_F - \beta s\phi + \pi(\phi)(1 - s\beta) \frac{s\rho + \gamma}{1 - \gamma^2} + \lambda_2,$$

where $\lambda_2$ is the Lagrange multiplier measuring the shadow costs of the constraint $(\phi - \phi) \geq 0$ and $\pi(\phi)$ is given by (3).

From the first partial derivative it is clear that the profits are maximized at $s = \frac{1}{\beta} \in [0,1]$. Given this value, the optimisation problem, including the optimal effort levels, is a replica of the optimisation problem solved when the type was verifiable. Its solution is then identical.

**Appendix 3: Proof of Proposition 4:**

We focus on the case where $\hat{\phi} < \bar{\phi}$. In the text we explain why $e^*_p \geq \sigma \bar{\phi}$ suffices when the firm targets a top scientist.

$(\Rightarrow)$ Assume that $e_p = \sigma \hat{\phi}$ and $w = w - e_p \hat{\phi} + C(e_F, e_p) + \frac{\sigma}{2} (\hat{\phi})^2$.

The participation constraint (17) clearly binds. The incentive constraint (18) holds provided

$$\sigma \hat{\phi}(\hat{\phi} - \phi') \geq \frac{1}{2} \sigma(\hat{\phi} - \phi')(\hat{\phi} + \phi').$$

It is straightforward to verify that the above holds whether $(\hat{\phi} - \phi') > 0$ or $(\hat{\phi} - \phi') < 0.$

$(\Leftarrow)$ Assume that the incentive constraint holds and that the participation constraint binds.

To extract all rents the firm must set $w = w - e_p \hat{\phi} + C(e_F, e_p) + \frac{\sigma}{2} (\hat{\phi})^2$. Given this wage, if the incentive constraint holds it means that

$$e_p(\hat{\phi} - \phi') \geq \frac{1}{2} \sigma(\hat{\phi} - \phi')(\hat{\phi} + \phi').$$
for any $\phi'$. This must particularly be true for $\phi' = \hat{\phi} + \varepsilon$ and $\phi' = \hat{\phi} - \varepsilon$ with $\varepsilon > 0$ and must hold as $\varepsilon \to 0$. Thus we must have

$$\hat{e}_p \leq \frac{1}{2} \sigma(2\hat{\phi} + \varepsilon) \text{ and } \hat{e}_p \geq \frac{1}{2} \sigma(2\hat{\phi} - \varepsilon).$$

Since $\lim_{\varepsilon \to 0} \frac{1}{2} \sigma(2\hat{\phi} + \varepsilon) = \lim_{\varepsilon \to 0} \frac{1}{2} \sigma(2\hat{\phi} - \varepsilon) = \sigma\hat{\phi}$, the two inequalities above can only hold simultaneously provided $\hat{e}_p = \sigma\hat{\phi}$. □

Appendix 4: Proof of Proposition 6:

Firm $i$ ($i = 1, 2$) selects a scientist of ability $\phi$ that maximizes (11) subject to the additional constraint $e_p(\phi) = \sigma\phi$. As we replace $e_p$ by $\sigma\phi$, the first order conditions with respect to $e_F$ and ability are given by

$$\frac{\partial \Pi(\phi)}{\partial e_F} = \pi(\phi) - \beta e_F + \gamma\beta\sigma\phi = 0$$

and

$$\frac{\partial \Pi(\phi)}{\partial \phi} = e_F\rho + \gamma\beta\sigma e_F - \beta\sigma\phi(\sigma - 1) + \lambda_2 = 0,$$

where $\lambda_2$ is the Lagrange multiplier measuring the shadow costs of the constraint $(\bar{\phi} - \phi) \geq 0$. From the first equation we get

$$\beta e_F = \tau + \phi(\rho + \gamma\beta\sigma) > 0.$$ 

Given these effort levels we have (after simplifications)

$$\frac{\partial \Pi}{\partial \phi} = \frac{1}{\beta}[\tau(\rho + \gamma\beta\sigma) - \delta^{**}\phi] - \lambda_2.$$

Where $\delta^{**} = \beta^2\sigma(\sigma - 1) - (\rho + \gamma\beta\sigma)^2$. There exists a unique positive value $\Sigma$ such that $\delta^{**} \leq 0$ for $\sigma \in [0, \Sigma]$ and $\delta^{**} \geq 0$ for $\sigma \geq \Sigma$.

When $\delta^{**} \leq 0$, the objective function is increasing with the type. Therefore $\lambda_2 > 0$ and it is optimal to set $\phi^{**} = \overline{\phi}$. When $\delta^{**} > 0$ there exists a potential interior solution $\phi^{**} = \frac{\tau(\rho + \gamma\beta\sigma)}{\delta^{**}}$. One can show that exists a unique positive value $T^{**}$ such that $\phi^{**} = \overline{\phi}$. Moreover $\phi^{**} < \overline{\phi} \iff \sigma > T^{**}$. Specifically we have
\[ T^{***} = \frac{\overline{\phi}(\beta + 2\gamma \rho) + \gamma + \sqrt{(\overline{\phi}(\beta + 2\gamma \rho) + \gamma)^2 + 4(\rho \overline{\phi})^2 (1 - \gamma^2)}}{2\beta \overline{\phi}(1 - \gamma^2)}. \]

Clearly, when \( \sigma = \Sigma \) we have \( \frac{\tau(\rho + \gamma \beta \sigma)}{\delta^{**}} > \overline{\phi} \) so that \( \Sigma < T^{***} \). Therefore the following applies:

- For any \( \sigma \in [0, T^{***}] \) it is optimal to set \( \phi^* = \overline{\phi} \) either because the objective function increases with the type or because \( \frac{\tau(\rho + \gamma \beta \sigma)}{\delta^{**}} > \overline{\phi} \).
- For any \( \sigma \geq T^{***} \) it is optimal to set \( \phi^{***} = \frac{\tau(\rho + \gamma \beta \sigma)}{\delta^{**}} \).
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