2010
18

Working Paper
INSTITUTO DE POLÍTICAS Y BIENES PÚBLICOS (IPP)

OPTIMAL CARBON SEQUESTRATION PATH WHEN DIFFERENT BIOLOGICAL OR PHYSICAL SEQUESTRATION FUNCTIONS ARE AVAILABLE

ALEJANDRO CAPARRÓS
CSIC-INSTITUTE OF PUBLIC GOODS AND POLICIES (IPP)

DAVID ZILBERMAN
UNIVERSITY OF CALIFORNIA AT BERKELEY
DEPARTMENT OF AGRICULTURAL AND RESOURCE ECONOMICS

CSIC
CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS
CCHS
Centro de Ciencias Humanas y Sociales
IPP
INSTITUTO DE POLÍTICAS Y BIENES PÚBLICOS

INSTITUTO DE POLÍTICAS Y BIENES PÚBLICOS CCHS-CSIC

Instituto de Políticas y Bienes Públicos
Centro de Ciencias Humanas y Sociales
Consejo Superior de Investigaciones Científicas
C/ Albasanz, 26-28.
28037 Madrid (España)

Tel: +34 91 602 2300
Fax: +34 91 304 5710

http://www.ipp.csic.es/

How to quote or cite this document:

Caparrós, A. & Zilberman, D. (2010). Optimal carbon sequestration path when di¤erent biological or physical sequestration functions are available. Instituto de Políticas y Bienes Públicos (IPP), CCHS-CSIC, Working paper, Number 18.
http://hdl.handle.net/10261/28913

2010
18
Working Paper
INSTITUTO DE POLÍTICAS Y BIENES PÚBLICOS (IPP)

OPTIMAL CARBON SEQUESTRATION PATH WHEN DIFFERENT BIOLOGICAL OR PHYSICAL SEQUESTRATION FUNCTIONS ARE AVAILABLE

ALEJANDRO CAPARRÓS
CSIC-INSTITUTE OF PUBLIC GOODS AND POLICIES (IPP)
ALEJANDRO.CAPARROS@CCHS.CSIC.ES

DAVID ZILBERMAN
UNIVERSITY OF CALIFORNIA AT BERKELEY
DEPARTMENT OF AGRICULTURAL AND RESOURCE ECONOMICS
ZILBER11@BERKELEY.EDU

Optimal carbon sequestration path when different biological or physical sequestration functions are available*

November 6, 2009

Alejandro Caparrós

Spanish National Research Council (CSIC), Institute for Public Goods and Policies (IPP), Albasanz 26, E-28037 Madrid (Spain). Tel. +34 91 6022536. Fax +34 91 6022971. E-mail: alejandro.caparros@cchs.csic.es. *Corresponding author.*

David Zilberman

Department of Agricultural and Resource Economics, University of California at Berkeley, Berkeley, CA 94720-3310, USA. Tel: (510) 642-6570. E-mail: zilber11@berkeley.edu

*We thank participants at the EAERE Workshop on Carbon Sequestration in Agriculture and Forestry (Thessalonica, Greece), the AERNA2008 Conference (Majorca, Spain), the FEEM Workshop on the Economics of Climate Change (Sardinia, Italy) and the REDE 3rd Atlantic Workshop on Energy and Environmental Economics (La Toja, Spain) for their helpful comments. Alejandro Caparrós was visiting UC Berkeley with a grant from the Spanish Ministry of Education while working on this paper. The usual disclaimer applies.

Optimal carbon sequestration path when different biological or physical sequestration functions are available

Abstract: We set out a general framework to discuss carbon sequestration programs when different alternatives are available and each of them yields sequestration benefits far into the future and at varying rates. We focus on reforestations, since trees grow for a long time, at varying rates, and different types of species yield completely different sequestration rates. We show that the Social Planner (and the landowner) will continuously change the species used for the reforestations and that the trend is to use slower and slower growing species as the land available for reforestations becomes scarcer and carbon builds up in the atmosphere.

JEL classification : C61, Q23, Q54.

Key words: carbon sequestration, forests, optimal control, vintage, Volterra integral.

1 Introduction

Carbon sequestration through land use changes and forestry has received considerable attention in the climate change literature, and it is now clear that any meaningful climate policy has to include these alternatives to provide a cost-effective mitigation portfolio. The Kyoto Protocol recognizes this (although leaving out, for the time being, avoided deforestation) and any future international or domestic effort to fight climate change will continue to do so. However, these alternatives imply particular land uses and are limited by their own nature. Hence, land use change and forestry options are widely seen as a means to "buy time" and should therefore be analyzed within the context of a dynamic general equilibrium model that allows for the competition with alternative abatement strategies. Setting aside the Integrated Assessment Models, where the forestry sector is treated as a separate module (e.g. Sohngen and Mendelsohn (2003) or Tavoni *et al.* (2007)), existing optimal control models that integrate carbon sequestration with other abatement measures use rather particular growth functions. The two models that are closer to ours assume either instantaneous growth (Feng *et al.*, 2002) or a simple discrete growth function (Caparrós, 2009). Ragot and Schubert (2008) focus on soil carbon sequestration using exponential functions to describe the process. Although not dealing with reforestations, this is probably the theoretical model that integrates the more realistic "growth function" in a dynamic general equilibrium model. However, they assume that in each period the Social Planner can only decide whether or not to change the use of a pre-defined amount of land (i.e. the decision variable can only take two possible values).

More importantly for our analysis, existing models fail to take into account that different types of species grow differently. This is the case in the dynamic general equilibrium models in Feng *et al.* (2002), Ragot and Schubert (2008) or Caparrós (2009), but also in most models that focus on the forest level (see, however, Caparrós *et al.* (2009) for a model with two species). Nevertheless, there are important questions that need to take into account the diversity of trees available and their different growth patterns. We might, for example, ask what is better, to plant fast growing species that will reach maturity relatively soon, or to plant slow growing species that will deliver lower amounts of sequestration per year initially but will continue to grow far into the future? Furthermore, does the answer to this question change as carbon stocks build up in the atmosphere?

To fill this gap, this paper presents a general framework that, within a dynamic general equilibrium model, allows the Social Planner (and/or the forestry owner) to choose the type of species to plant and that, in each moment in time, takes into account the growth performed by all the trees planted in the past. Although we focus throughout the exposition on reforestations, we have kept the model general enough to be able to deal with practices that enhance soil carbon sequestration, or with any other measure that yields carbon sequestration benefits for several years at a varying rate. In fact, from a modeling point of view carbon sequestration in trees or in soils is relatively similar, since both yield higher initial rates of sequestration that tend to decrease until a saturation point is reached (see van't Veld and Plantinga (2005) for forests and Watson *et al.* (2000) for soil or sinks in other terrestrial ecosystems).

The main feature of our model is that each portion of forest planted at any period s previous to t continues to grow at a rate that depends on the type of species chosen at time s and on the age of the forest (the time elapsed since s at period t). This implies having an optimal control problem with an integral state equation since with general functions this integral is of the Volterra type, and can thus not be simplified to the standard optimal control problem by taking the time derivative (Vinokurov, 1969; Kamien and Muller, 1976). The main implication of this type of state equations is that they allow the aggregation over time of the contribution of assets of different vintages.

In economics, Volterra state-equations have been used mainly to analyze durable goods (Schmalensee, 1979; Muller and Peles, 1990) in so called "vintage" models. The assumption made in this literature that the object produced is not sold but that the producer retains the property and gets an annual payment is rather artificial in the case of a durable good such as a car but is natural in the case of a payment for the amount of carbon sequestered. Technically, however, the two approaches are relatively similar (although we do not only assume different decay rates but also different initial performance for different types of trees, and we have to take into account land availability).

In forestry economics, the term "vintage" model has been used in studies that take into account that trees grow differently depending on their age, but which consider that only one type of tree is available (see Salo and Tahvonen (2002), or Costa-Duarte *et al.* (2006) for an application to carbon sequestration issues). The similarities of our analysis with this branch of the literature are rather small, since we consider not only differences in growth due to age but also differences due to the different species used in the past for reforestations. In addition, our analysis is done within a general equilibrium model where atmospheric carbon stocks are taken into account.

Finally, state equations with integrals also appear in another strand of the literature that solves quality distributed-intertemporal optimal control problems. This methodology allows considering, e.g., quality–specific abatement strategies (Xabadia et al., 2006) or the management of size-distributed forests (Calvo-Calzada and Goetz, 2001). The main difference with our approach is that in these models the state equation integrates over qualities, not over time, so that all the relevant information is summarized in variables evaluated at time t.

As in Feng et al. (2002), we show that sequestration is a transitory policy leading to a steady-state where no additional reforestations occur. Thus, the challenge of the optimization is to manage the transition. In this regard, we show that the optimal policy is to start planting fast growing species and to use slower and slower growing species as time goes by and carbon builds up in the atmosphere. As should be expected, the amount of land reforested also declines over time until the steady-state is reached. We also find the institutions, i.e. the implementation mechanisms, which ensure that the socially optimal path, in terms of species chosen and amount of land converted, is followed by the private landowners.

However, since analyzing the approach path for the full model is difficult we first analyze graphically the carbon emission and stock phase-diagram to obtain the qualitative behavior of the carbon price. Taking this carbon price path as given, we then show that if the implementation of the carbon sequestration program is done using the Carbon Flow Method (van Kooten *et al.*, 1995) the landowner follows the same path than the one that the Social Planner would follow within the general model (we discuss three alternative carbon accounting methods in Section 4). Finally, we analyze the optimal sequestration path in the nested model (the problem for the landowner), showing that the incentives to use fast growing species decrease over time and that the land converted also declines over time. That is, as land becomes scarcer and scarcer, the Social Planner (and/or the forest owner) will plant less and less but will use species that will continue to provide benefits further and further into the future. This result implies that postulating that only one species exist is a very strong assumption, since the decision maker will only choose the same species in two subsequent periods at the steady-state. In addition, at the steady-state no reforestation is done in the first place, so that choosing the same species has little impact.

2 The model

A society is emitting $e(t)$ units of carbon at time t. The benefits from emissions that determine the demand for carbon emissions are $B(e(t))$, with $B(0) = 0$, $B_e > 0$, $B_{ee} \leq 0$ and $\lim_{e \to \infty} B(\cdot) = \infty$. Damage caused by carbon stored in the atmosphere $C(t)$ is denoted by $D(C(t))$, with $D(0) = 0$, $D_C > 0$, and $D_{CC} > 0$. Atmospheric carbon content decays naturally (σ) and can be reduced through land use changes, such as reforestations or soil carbon sequestration practices. The opportunity cost of these activities depends on their acreage. We call $A(t)$ the total units of land enrolled in carbon sequestration programs at time t, with $A(0) = 0$, and $Q(A(t))$ the net opportunity cost of enrolling these units of land in the sequestration program instead of devoting them to agriculture or the most profitable alternative use, with $Q(0) = 0$, $Q_A > 0$, $Q_{AA} > 0$ and $\lim_{A \to \bar{A}} Q(\cdot) = \infty$, where $\bar{A}$ is the total land available (this assumption ensures that the last units of land will never be converted). The amount of land that is enrolled in a carbon sequestration program at time t is called $a(t)$. This can refer, e.g., to a reforestation program or to a soil carbon sequestration program in agricultural land (to simplify the exposition we will refer from now on primarily to reforestations).

The land enrolled in a carbon sequestration program will sequester carbon at any time

$t \geq s$ at a decreasing rate from the moment s when it was originally converted (when the forest was planted). The type of species chosen at time s is called $b(s)$ and the growth at any moment $t \geq s$ is given by $g(t, b(s), s)$, with $g(s, b(s), s) \geq g(t, b(s), s)$ for all $t \geq s$ (i.e. the initial growth rate is the highest). We assume a one to one relation between growth and carbon sequestration[1] and that an increase in $b(s)$ implies faster growing species. We define a faster growing species as one that has initially higher growth rates, although future growth rates may or may not be higher than for slower growing species. That is, we have that $g_b(s, b(s), s) > 0$, but not necessarily that $g_b(t, b(s), s) > 0$ for all $t \geq s$. We are thus assuming the existence of a continuum of species each one with its particular growth function. This assumption is only partially realistic since the different species available would be best represented by a *piecewise* continuous function. Changing from one species to another implies a jump, but each species has a large number of varieties that can be selected according to their fit to specific niches. The number of possible variations within one species (or sub-species) is so large that it is best represented by a continuum[2]. However, although we acknowledge that between species there might be a jump we rule out step discontinuities to facilitate the analysis (our qualitative results would probably not change if this assumption were relaxed).

We start from a situation where $A(0) = 0$ and focus on the part of the problem where $a(s) \geq 0$. In other words, as in van't Veld and Plantinga (2005) we assume that only permanent forests can be established, i.e. we do not consider the possibility of reforesting first

[1] Relaxing this assumption would just add parameters to the model without any real gain.

[2] "[Biological] diversity is often represented as a hierarchy of discrete units [... but] it is understood that biological diversity is really more a continuum – particularly with plant species that tend to hybridize more freely than do animals. The discrete units are helpful organizing tools but to truly understand biological diversity, the continuity should be considered. [...] So genetic diversity neither ends nor begins with the species: it continues in both directions." (NFGEL and GRCP, 2006).

and deforesting afterwards[3]. The permanent forest established can be un-managed or managed, but we assume that if managed the same species will re-grow, either by natural or by artificial regeneration, and managed to reach a "normal" forest structure. For a justification of this assumption see van't Veld and Plantinga (2005), who argue further that both, the growth of a managed or of an un-managed forest with replacement, can be approximated using exponential functions. We will use more general functions (except in the Appendix) but assuming nevertheless that the permanent forest will ultimately yield a steady-state where $g(t, b(s), s) = 0$ when $t \to \infty$. This implies that the total amount of carbon (biomass) in one particular hectare will reach a maximum $M(b(s))$ which depends on the particular species.

At any moment of time t the flow of carbon sequestered (total growth) is given by equation (3), see below. That is, each portion of forest planted at any period s previous to t continues to grow at a rate that depends on the type of species chosen at time s and on the age of the forest (the time elapsed since s at period t). This implies having an optimal control problem with an integral state equation (see Kamien and Muller (1976); or Vinokurov (1969) or Bakke (1974) for a more rigorous analysis). This integral is of the Volterra type and it cannot be simplified to the standard state equation by taking the time derivative.

The net cost of enrolling (planting) $a(t)$ units of land with species $b(t)$ is denoted by $R(a(t), b(t))$, with[4] $R_a > 0$, $R_{aa} > 0$, and $R_{ab} = 0$. Finding a clear relationship for R_b and R_{bb} is difficult. For one particular species increasing the growth rate implies higher costs

[3] Although their results are not directly applicable since they use different models, Feng et al. (2002) and Ragot and Schubert (2008) show that reforesting first and deforesting afterwards is not an optimal strategy.

[4] Increasing the units of land reforested in a given year increases costs more than proportionallly, e.g. as specialized labor becomes scarce, salaries increase (see van Kooten (2000)). Setting reasonable assumptions for the cross derivatives is a complicated task. The increase in the marginal reforestations costs will probably be smaller when different species are available since the most appropriate ones can be choosen for different areas, but since this impact is probably small we assume that the cross derivaties are equal to zero.

since irrigation or fertilization is needed (i.e. for the continuous part of the ideal piecewise function described above), but changing to a faster growing species may increase the costs, but may also reduce them in some areas. Nevertheless, the general tendency seems to go in the direction of an increase in reforestation costs for faster growing species[5]. Thus, we assume that $R_b > 0$ and $R_{bb} > 0$ holds.

To focus the exposition on carbon sequestration we assume that no additional benefits accrue to society (or to the landowner) from the land devoted to the carbon program. The Social Planner's (SP) problem is (r is the discount rate and T may be taken to be finite or infinite, with the appropriate transversality conditions):

$$\max_{e(t),a(t),b(t)} \int_0^T e^{-rt}[B(e(t)) - D(C(t)) - Q(A(t)) - R(a(t),b(t))]dt \tag{1}$$

$$\dot{C}(t) = e(t) - \sigma C(t) - G(t) \tag{2}$$

$$G(t) = \int_0^t a(s)g(t,b(s),s)ds \tag{3}$$

$$\dot{A}(t) = a(t) \tag{4}$$

[5] We used the coefficients reported by Sohngen for the Global Timber Market and Forestry Data Project (Version 5, 2007, see http://aede.osu.edu/people/sohngen.1/forests/GTM/index.htm) to fit two simple functions explaining regeneration costs as a function of yield after 10 years using data for the US and for Canada (we used yield after 10 years as a proxy for initial growth since most of the empirical growth functions estimated by Sohngen yield negative values for $t = 1$). Since our definition of a faster growing species implies that initial growth is larger, an increase in yield at year 10 by passing from one species to another should imply larger reforestation costs if $R_b > 0$ (Sohngen actually refers to regeneration costs). The estimated functions have the form: $RC_i = x_{1i}e^{x_{2i}Y_{10i}}$, where $i = \{US, Canada\}$, RC = regeneration costs, Y_{10} = yield at year 10, and $\{x_1, x_2\}$ are parameters. In both cases the estimated values for x_1 and x_2 are positive (although R^2 values are low, especially for the US). This implies $R_b > 0$ and $R_{bb} > 0$. In addition, Caparrós et al. (2009), in their empirical application to the South of Spain, also find that the slow growing species available (cork oak) has lower reforestation costs than the fast growing species available (eucalyptus). All this is only weak evidence but it should suffice to justify the assumption mentioned in the main text in a highly stylized model as the one presented here.

The Hamiltonian and the necessary conditions are as follows:

$$H_0 = e^{-rt}[B(e(t)) - D(C(t)) - Q(A(t)) - R(a(t), b(t)]$$
$$+\lambda(t)\left[e(t) - \sigma C(t) - G(t)\right] + \int_t^T a(t) g(s, b(t), t)\mu(s) ds + \varphi(t)\left[a(t)\right]$$

$$\frac{\partial H_0}{\partial e} = e^{-rt} B_e(e(t)) + \lambda(t) = 0 \tag{5}$$

$$\frac{\partial H_0}{\partial a} = -e^{-rt} R_a(a(t), b(t)) + \int_t^T g(s, b(t), t)\mu(s) ds + \varphi(t) = 0 \tag{6}$$

$$\frac{\partial H_0}{\partial b} = -e^{-rt} R_b(a(t), b(t)) + \int_t^T a(t) g_b(s, b(t), t)\mu(s) ds = 0 \tag{7}$$

$$\dot{\lambda}(t) = -\frac{\partial H_0}{\partial C} = e^{-rt} D_c(C(t)) + \sigma\lambda(t) \tag{8}$$

$$\mu(t) = \frac{\partial H_0}{\partial G} = -\lambda(t) \tag{9}$$

$$\dot{\varphi}(t) = -\frac{\partial H_0}{\partial A} = e^{-rt} Q_A(A(t)) \tag{10}$$

Substituting and rearranging we get:

$$R_a(a(t), b(t)) = \int_t^T e^{-r(s-t)} g(s, b(t), t) B_e(e(s)) ds - \int_t^T e^{-r(s-t)} Q_A(A(s)) ds \tag{11}$$

$$R_b(a(t), b(t)) = \int_t^T e^{-r(s-t)} a(t) g_b(s, b(t), t) B_e(e(s)) ds \tag{12}$$

Condition (11) requires that the increase in marginal planting cost associated with one additional unit of land equals the future stream of carbon benefits associated with the reforestation (first integral) minus the future stream of opportunity costs. The future stream of benefits of the carbon sequestration is valued using the "price" for carbon given by the benefit that accrues to society from the emission of one unit of carbon. Condition (12) requires the increase in planting costs incurred by choosing a faster growing species to be equal to the value of the increase in future sequestration obtained by this change of species.

From (4) we know that at the steady-state $a^* = 0$ and this implies, in turn, that sequestration at the steady-state is also zero ($G^* = 0$ when $t \to \infty$). That is, as in Feng et al. (2002), the steady-state is of limited interest in our model since no reforestations occur (we nevertheless derive the steady-state values in the Appendix). However, long after the steady-state for this variable is reached G will continue to be positive since all the reforestations previously done will continue to yield carbon sequestration benefits. That is, although strictly speaking carbon sequestration is only a temporary solution, if species are chosen (especially for the last reforestations) where growth decays slowly these reforestations may have an impact on climate for a very long period (what enters in equation (2) is G and not a). In fact, this period may be of several hundreds years for some tree species.

Analyzing the optimal path for the complete model is difficult, since we have three control variables and three state variables (one of the Volterra type). We therefore decompose the problem in two parts. First, we analyze graphically the optimal path of emissions. We then focus on the optimal path for the variables directly related to carbon sequestration (the rate of land conversion and the species selection).

To draw the (quasi)-phase diagram in the $\{e(t), C(t)\}$-space set $\dot{C} = 0$ in (2) to get $e(t) = \sigma C(t) + G(t)$. Thus, the $\dot{C} = 0$ locus is linear and upward sloping, with its location depending on the value of $G(t)$. For the equation of motion of $e(t)$ differentiate equation (5) and plug it in (8) together with (5) to obtain:

$$\dot{e}(t) = \frac{(r + \sigma)\, B_e(e(t)) - D_c(C(t))}{r B_{ee}(e(t))}$$

Setting $\dot{e}(t) = 0$ we get $D_c(C(t)) = \sigma B_e(e(t))$. By totally differentiating we obtain:

$$\frac{de(t)}{dC(t)} = \frac{D_{cc}(C(t))}{(r+\sigma)\, B_{ee}(e(t))} < 0$$

Hence, the locus is downward sloping and independent of $G(t)$. Figure 1 shows the (quasi)-phase diagram in the $\{e(t), C(t)\}$-space. The most reasonable initial point for the climate change problem is north-west of the steady-state given by (e^*, C^*). Atmospheric CO_2 concentrations have been increasing since pre-industrial levels and are expected to increase further before a new steady-state is reached, and all the evidence available suggests that the current level of emissions has to be reduced in the coming decades to reach an optimal (reasonable) new steady-state (IPCC, 2007; Feng *et al.,* 2002). As the streamlines in figure 1 show, without carbon sequestration the system would approach (e^*, C^*) following a stable branch towards the saddle-point equilibrium (as long as the initial point lies below the $\dot{e}(t) = 0$ isocline). The existence of carbon sequestration will initially move the $\dot{C} = 0$ isocline to the left as $G(t) > 0$ increases (see (2)) and then shift it back to the location of the $\dot{C} = 0$ isocline when $G(t) = 0$ (we have shown above that $G(t) = 0$ when $t \to \infty$). As long as growth in forests is always lower than emissions minus natural carbon decay, $G(t) \leq e(t) - \sigma C(t)$, the system does not cross the $\dot{C} = 0$ isocline and approaches the steady-state following a path above the path without sinks but within the stable branch shown in figure 1 for the $\{e(t), C(t)\}$-space. This is not the only possible path in our general model, but it is the more reasonable one for the climate problem under consideration. In fact, van't Veld and Plantinga (2005) have shown in an empirical model that even very large reforestation programs have very little effect on the carbon price path. This implies, in our framework, that the $\dot{C} = 0$ isocline will only be moved slightly to the left, and that the emissions path will only be slightly above the path without sinks. In any case, with or without carbon sequestration, emissions will have to decrease in the coming decades and, given our assumptions on $B_e(e(t))$, this implies increasing marginal benefits of emissions, until the steady-state value $B_e(e^*)$ is reached. If we further assume that an efficient carbon trading market is in place (see Feng *et al.* (2002) for a justification), so that $P(t) = B_e(e(t)) = -\lambda(t)e^{rt}$, we know that the qualitative behavior of the carbon price will be to increase over time until the equilibrium price $P^* = B_e(e^*)$ is reached. Using this qualitative path for carbon prices, in the next section we investigate the optimal path for the rate of land conversion and for the species selection, by focusing on the problem for the landowner. We first show that, if the implementation mechanism is well defined, the problem for the landowner is exactly the same as the part of the overall problem for the SP that determines the optimal choice of land enrolled ($a(t)$) and species used ($b(t)$), for a given carbon price path. Thus, by analyzing the optimal behavior of the landowner we obtain the optimal path that the SP would choose for $a(t)$ and $b(t)$.

[Figure 1]

3 Optimal path of sequestration: the nested model

From the several carbon accounting methods proposed in the literature we focus in this section on the 'Carbon Flow Method' (CFM), leaving the analysis of alternative carbon accounting methods for the next section. The CFM was proposed in the early literature on the impact of carbon sequestration on optimal rotations (Englin and Callaway (1993) or van Kooten *et al.* (1995)) and essentially implies that the land owner (or forest owner) gets paid when carbon sequestration takes place and has to pay when carbon is released. The amount to be paid is set equal to the carbon price associated with CO_2 emissions. The payment can come from the government (with a subsidy for sequestration and a tax on liberation) or from an efficient carbon trading system. Van't Veld and Plantinga (2005) use this method and Feng *et al.* (2002) call it 'pay-as-you-go', although in their case it is not clear whether the payment occurs for carbon sequestration or for land conversion (since both are synonymous in their model). This is a reasonable incentive mechanism to be set up by Annex-I governments within the Kyoto framework since what counts at the international level is the total carbon budget of the country, or more precisely the average during the commitment period (Caparrós and Jacquemont, 2003). Richards *et al.* (2006) also present this method as one of the best alternatives to be used in the United States.

We first show that the CFM is efficient, in the sense that the landowner would follow the same optimal path on his land than the one that the Social Planner would (same amount of reforestations each year and using the same species), given a path for carbon prices.

The problem for a landowner under the CFM is:

$$\max_{a(t),b(t)} \int_0^T e^{-rt}[P(t)G(t) - Q(A(t)) - R(a(t), b(t))]dt \tag{13}$$

$$G(t) = \int_0^t a(s)g(t, b(s), s)ds \tag{14}$$

$$\dot{A}(t) = a(t) \tag{15}$$

where $P(t)$ is the price of carbon, which we assume to be set by an efficient emission trading system, i.e., the carbon price is given for the landowner (see the discussion above). Forming the Hamiltonian the necessary conditions are:

$$
\begin{aligned}
\frac{\partial H}{\partial a} &= -e^{-rt}R_a(a(t),b(t)) + \int_t^T g(s,b(t),t)\mu(s)ds + \varphi(t) = 0 \qquad (16)\\
\frac{\partial H}{\partial b} &= -e^{-rt}R_b(a(t),b(t)) + \int_t^T a(t)g_b(s,b(t),t)\mu(s)ds = 0 \qquad (17)\\
\mu(t) &= \frac{\partial H}{\partial G} = e^{-rt}P(t) \qquad (18)\\
\dot{\varphi}(t) &= -\frac{\partial H}{\partial A} = e^{-rt}Q_A(A(t)) \qquad (19)
\end{aligned}
$$

Substituting and rearranging we get:

$$
\begin{aligned}
R_a(a(t),b(t)) &= \int_t^T e^{-r(s-t)}g(s,b(t),t)P(s)ds - \int_t^T e^{-r(s-t)}Q_A(A(s))ds \qquad (20)\\
R_b(a(t),b(t)) &= \int_t^T e^{-r(s-t)}a(t)g_b(s,b(t),t)P(s)ds \qquad (21)
\end{aligned}
$$

These conditions are exactly the same as conditions (11) - (12), just substituting $B(e(s))$ by $P(s)$. Thus, as long as the emission trading system is efficient, *i.e.* $P(t) = B_e(e(t)) = -\lambda(t)e^{rt}$ (or the SP chooses the carbon price accordingly) the optimal paths that we are going to characterize for $a(t)$ and $b(t)$ are the same in the nested model (landowner) as in the general model (SP). Thus, we can write the following lemma:

Lemma 1 *Assume that an efficient carbon price exists in the economy* ($P(t) = B_e(e(t)) = -\lambda(t)e^{rt}$). *Then the CFM is efficient, in the sense that the Social Planner and the landowner would reforest each year the same amount of land using the same species.*

Substituting $\mu(t)$ (from ((18)), taking the time derivative of (16) and (17) and finally substituting $\dot{\varphi}(t)$ (from ((19)) we have:

$$\begin{aligned} 0 &= re^{-rt}R_a - e^{-rt}R_{aa}\dot{a}(t) - e^{-rt}R_{ab}\dot{b}(t) + \dot{b}(t)\int_t^T g_b(s,b(t),t)e^{-rs}P(s)ds \\ &\quad -g(t,b(t),t)e^{-rt}P(t) + e^{-rt}Q_A \end{aligned} \tag{22}$$

$$\begin{aligned} 0 &= re^{-rt}R_b - e^{-rt}R_{ba}\dot{a}(t) - e^{-rt}R_{bb}\dot{b}(t) + \dot{b}(t)\int_t^T a(t)g_{bb}(s,b(t),t)e^{-rs}P(s)ds \\ &\quad -a(t)g_b(t,b(t),t)e^{-rt}P(t) + \dot{a}(t)\int_t^T g_b(s,b(t),t)e^{-rs}P(s)ds \end{aligned} \tag{23}$$

Since we know that

$$\begin{aligned} H_{aa} &= -e^{-rt}R_{aa} \\ H_{ab} &= H_{ba} = -e^{-rt}R_{ab} + \int_t^T g_b(s,b(t),t)e^{-rs}P(s)ds \\ H_{bb} &= -e^{-rt}R_{bb} + \int_t^T a(t)g_{bb}(s,b(t),t)e^{-rs}P(s)ds \end{aligned}$$

equations (22) and (23) simplify to

$$H_{aa}\dot{a}(t) + H_{ab}\dot{b}(t) = e^{-rt}\left[g(t,b(t),t)P(t) - (Q_A + rR_a)\right] = X_1 \tag{24}$$

$$H_{ab}\dot{a}(t) + H_{bb}\dot{b}(t) = e^{-rt}\left[a(t)g_b(t,b(t),t)P(t) - rR_b\right] = X_2 \tag{25}$$

Solving for the two unknowns:

$$\dot{a}(t) = \left[X_1H_{bb} - X_2H_{ab}\right]\left[H_{aa}H_{bb} - (H_{ab})^2\right]^{-1} \tag{26a}$$

$$\dot{b}(t) = \left[X_2H_{aa} - X_1H_{ab}\right]\left[H_{aa}H_{bb} - (H_{ab})^2\right]^{-1} \tag{26b}$$

Assuming an interior solution we know that the Hessian matrix of the Hamiltonian is negative semidefinite, and this implies that $H_{aa} < 0$, $H_{bb} < 0$ and $H_{aa}H_{bb} > (H_{ab})^2$. In addition, since $R_{ab} = 0$ (this would also hold for $R_{ab} \leq 0$), H_{ab} is positive if (28) holds. To have $X_1 > 0$, we need the term in brackets in (24) to be positive. Substituting equation (20) we get:

$$\frac{g(t, b(t), t)P(t)}{r} - \frac{Q_A(A(t))}{r} > \int_t^T e^{-r(s-t)} g(s, b(t), t)P(s)ds - \int_t^T Q_A(A(t))e^{-r(s-t)}ds \quad (27)$$

Given our assumptions on Q_A, Q_{AA}, and $a(t)$ we know that $Q_A(A(t)) \leq Q_A(A(s)) \quad \nabla s \geq t$ and therefore that the second term on the LHS of (27) is smaller than the second term on the RHS for $T = \infty$. Thus, condition (29) is a sufficient condition for (27) to hold.

To have $X_2 > 0$ we need the term in brackets in (25) to be positive. Substituting equation (21) we obtain (30). If these conditions hold, we get $\dot{a}(t) < 0$ and $\dot{b}(t) < 0$. That is, on the optimal path, we can write the following proposition:

Proposition 2 *Assuming $T = \infty$ and that an interior solution exists, if*

$$\int_t^T g_b(s, b(t), t)e^{-rs}P(s)ds > 0 \quad (28)$$

$$\frac{g(t, b(t), t)P(t)}{r} > \int_t^T e^{-r(s-t)} g(s, b(t), t)P(s)ds \quad (29)$$

$$\frac{g_b(t, b(t), t)P(t)}{r} > \int_t^T e^{-r(s-t)} g_b(s, b(t), t)P(s)ds \quad (30)$$

then $\dot{a}(t)$ is negative and $\dot{b}(t)$ is negative. That is, as long as the carbon price increase does not compensate the decrease in g and g_b, reforestations decline over time and slower and slower growing species are chosen.

The conditions $g_b(t, b(t), t) > 0$ (see the definition of fast growing species above[6]) and (28) ensure that by increasing $b(s)$ we are increasing sequestration not only in the first year but also that the increase in carbon sequestration over the entire period (valued at $P(s)$ and discounted) is positive. What matters is the discounted stream of the changes in growth (g_b) induced by choosing a faster growing species (valued at the future carbon price $P(s)$, assumed to be positive). In general this discounted stream will be larger for fast growing species (at least for high discount factors, where the results in the first years are paramount). Conditions (29) and (30) require that the increase in carbon prices should not compensate the decrease in g and in g_b. In fact, since the LHSs of equations (29) and (30) are equivalent to an infinite stream of constant terms, a sufficient condition for these two conditions to hold is that $g(s, b(t), t)P(s) \leq g(t, b(t), t)P(t)$ for all $s > t$ and that $g_b(s, b(t), t)P(s) \leq g_b(t, b(t), t)P(t)$ for all $s > t$. That is, as long as the initial growth (respectively, the marginal impact of a faster growing species) at the initial time (valued at the ongoing price) is larger than the value of any future growth rate (at the future price), the condition holds. If $P(t) = P$ (constant), a standard assumption in many carbon sequestration analyses as pointed out by van't Veld and Plantinga (2005), the first condition always holds since $g(s, b(t), t) \leq g(t, b(t), t)$ for all $s \geq t$, and it is reasonable to assume that $g_b(s, b(t), t) \leq g_b(t, b(t), t)$ for all $s \geq t$ also holds.

Maybe the most important consequence of Proposition 2 is that the landowner (and the Social Planner) will continually change the species used, except at the steady-state, where no reforestations take place anyway. Hence, a model that only assumes one type of species misses an important part of the story. In addition, if the conditions above are met, the tendency is to use slower and slower growing species as the land available for reforestations

[6] Note that s and t have changed their role while defining the Hamiltonian.

becomes scarcer and the last reforestations will therefore be done using very slow growing species so that $G(t)$ will remain positive long after $a(t)$ is set equal to zero. Thus, carbon sequestration programs will impact atmospheric carbon stocks for a long period.

4 Alternative carbon accounting methods

Most of the carbon accounting methods proposed in the literature can be classified into three major types, payment for carbon growth, payment for the stock of carbon and payment for land conversion. The CFM discussed in the previous section falls into the first category. We now discuss one additional method of the first type and one method for each one of the other two categories, linking these methods to similar methods proposed in the literature and to the rules within the Kyoto framework. We assume in all cases that an efficient emission trading system is in place that sets the price for carbon such that $P(t) = B_e(e(t))$.

The Carbon Annuity Account (CAA) method, proposed in Feng *et al.* (2002), has received less attention than the CFM although it is potentially an interesting method. Similar to the CFM, the generator of the carbon sequestration is paid the full value of the carbon emission price (the full value of a permanent reduction). However, instead of being paid to the forest owner it is put directly into an annuity account. As long as the carbon remains in place, the owner can access the earning of the annuity account but not the principal. When the carbon is released, the principal is reduced at the on-going carbon emissions permit price.

There are a number of methods that propose to pay for the carbon stock and not for the carbon flow. The carbon 'rental fee' used in Sohngen and Mendelsohn (2003) is such a method, where the forest owner gets paid a fee for each ton of carbon stored, for one year. Under the Variable Length Contract (VLC) method proposed in Feng *et al.* (2002) the forest owner also gets paid a smaller amount for each ton of carbon sequestered for a given period of time. However, as said above, enrollment of land and sequestration of the full amount of carbon go hand in hand in their model, so that their concept can be applied to both, to carbon sequestered or to land enrolled (see below). Another variation of this method is known as the 'ton-year-accounting-method' (see Moura-Costa and Wilson (2000)). With this method, the sequestration period considered is always one year, as in the 'rental fee' method, but instead of reducing the price to be paid what is reduced is the quantity of carbon credited[7]. The carbon accounting methods for the Clean Development Mechanism included in the Marrakech Accords (an agreement that completes the Kyoto Protocol) also pays for stock, setting the time period equal to 5 years for the t-CER method and equal to 30 years for the l-CER (Olschewski and Benítez, 2005). We will refer below to all these methods as Payment for the Stock of Carbon (PSC) and will assume that the payment is instantaneous, that is, under this method the landowner would get a constant stream of income related to the stock of carbon that he has on his fields.

Stavins (1999) or Lubowski *et al.* (2006) propose a Land Conversion Subsidy (LCS) for the conversion of land to forest and a tax on the conversion of land out of forest. A second feature of the policy is a requirement that afforested lands remain as forest for a specified period of time. As stated by Lubowski *et al.* (2006), this method is similar to the Conservation Reserve Program (CRP) in the United States, established by the Food Security Act of 1985, which provides annual rental payments to landowners voluntarily retiring environmentally

[7]Reduced by an equivalence factor that captures the benefit associated with sequestering one ton of CO_2 in the forest biomass for one year; this equivalence factor is estimated based on the cumulative radiative forcing of an emission of CO_2 over a 100-year time horizon.

sensitive land from crop production under 10- to 15-year contracts. Thus, this method is a reasonable way of encouraging carbon sequestration in practical terms, especially in the United States, giving the experience gained with the CRP program.

As the following proposition shows, all these methods can be made efficient, in the sense that the Social Planner and the landowner would reforest the same amount of land in any moment in time and with the same species. While the CFM and the CAA are efficient using $P(t)$ as the carbon price (as already shown in their framework by Feng et al. (2002)), the other two methods need to transform this price for carbon to be efficient. The condition shown below for the PSC implies that the amount to be paid per ton of carbon stock has to be smaller than the amount paid per ton of carbon flow (as should be expected). More importantly, the transformation of $P(t)$ proposed for the PSC does not need forward looking values. On the contrary, the condition shown for the LCS does indeed need future values.

Proposition 3 *Assume that an efficient carbon price exists in the economy* ($P(t) = B_e(e(t)) = -\lambda(t)e^{rt}$). *Then:*

(i) The CFM is efficient.

(ii) The CAA is efficient.

(iii) The PSC is efficient if the price per unit of carbon stock sequestered is set equal to:

$$q(t) = P(t)\frac{G(t)}{\int_0^t G(t)dt} \tag{31}$$

(iv) The LCS is efficient if $T = \infty$ *and the price per unit of land converted, and main-*

tained in the new use for τ periods, is set equal to:

$$m(t,\tau) = P(t)\frac{\tilde{G}(t)}{\tilde{a}(t)} - e^{-r\tau}P(t+\tau)\frac{\tilde{G}(t+\tau)}{\tilde{a}(t+\tau)} \tag{32}$$

Proof. (i) Direct from Lemma 1.

(ii) With the CAA, the landowner's problem is:

$$\begin{aligned} & \max \int_t^T e^{-rt}[M(t)r - Q(A(t)) - R(a(t),b(t))]dt \\ G(t) &= \int_0^t a(s)g(t,b(s),s)ds \\ \dot{A}(t) &= a(t) \\ \dot{M}(t) &= P(t)G(t) \end{aligned}$$

Forming the Hamiltonian the necessary conditions are:

$$\begin{aligned} \frac{\partial H}{\partial a} &= -e^{-rt}R_a(a(t),b(t)) + \int_t^T g(s,b(t),t)\mu(s)ds + \varphi(t) = 0 \\ \frac{\partial H}{\partial b} &= -e^{-rt}R_b(a(t),b(t)) + \int_t^T a(t)g_b(s,b(t),t)\mu(s)ds = 0 \\ \mu(t) &= \frac{\partial H}{\partial G} = \psi(t)P(t) \\ \dot{\varphi}(t) &= -\frac{\partial H}{\partial A} = e^{-rt}Q_A(A(t)) \\ \dot{\psi} &= -\frac{\partial H}{\partial M} = -re^{-rt} \end{aligned}$$

From $\dot{\psi} = -re^{-rt}$, we know $\psi = e^{-rt}$, and substituting we get (18). The remaining conditions are as in the CFM, thus, the CAA is efficient by Lemma 1.

(iii) The problem for the landowner under the PSC is:

$$\max_{a(t),b(t)} \int_0^\infty e^{-rt}[q(t)K(t) - Q(A(t)) - R(a(t), b(t))]dt \tag{33}$$

$$G(t) = \int_0^t a(s)g(t, b(s), s)ds \tag{34}$$

$$\dot{A}(t) = a(t) \tag{35}$$

where he gets paid $q(t)$ for the stock of all the growth that has taken place $K(t)$, which is defined as:

$$K(t) = \int_0^t \int_0^t a(s)g(t, b(s), s)dsdt = \int_0^t G(t)dt$$

Substituting (31) in (33) yields the objective function in (13). Hence, the problem becomes the same as the one for the CFM and we can apply Lemma 1 to show that the PSC is efficient under the condition shown.

(iv) To analyze the LCE landowner's problem define the land maintained in the program for τ units of time as:

$$\tilde{a}(t) \equiv \int_0^\infty a(t, \tau)d\tau - \int_0^t a(t - \tau, \tau)d\tau$$

The problem is then:

$$\max_{a(t,\tau)} \int_0^\infty \left[\int_0^\infty m(t, \tau)a(t, \tau)d\tau - Q(A(t)) - R(a(t), b(t))\right] e^{-rt}dt$$

$$\tilde{G}(t) = \int_0^t \tilde{a}(s)g(t, b(s), s)ds$$

$$\dot{A}(t) = \tilde{a}(t)$$

where $m(t,\tau)$ is the amount paid for maintaining a unit of land for τ units of time in the LCE program. Plugging the condition for efficiency in (32) into the objective function yields:

$$\begin{aligned}
&\int_0^\infty \left[\int_0^\infty m(t,\tau)a(t,\tau)d\tau - Q(A(t)) - R(a(t),b(t))\right] e^{-rt}dt \\
= &-\int_0^\infty \left[Q(A(t)) - R(a(t),b(t))\right] e^{-rt}dt + \int_0^\infty \left[\int_0^\infty P(t)\frac{\tilde{G}(t)}{\tilde{a}(t)}a(t,\tau)d\tau\right] e^{-rt}dt \\
&-\int_0^\infty \left[\int_0^\infty e^{-r\tau}P(t+\tau)\frac{\tilde{G}(t+\tau)}{\tilde{a}(t+\tau)}a(t,\tau)d\tau\right] e^{-rt}dt
\end{aligned}$$

The third term can be rearranged as follows (using the change of variable $x = t+\tau$, $y = \tau$ proposed by Feng *et al.* (2002)):

$$\begin{aligned}
\int_0^\infty \left[\int_0^\infty e^{-r\tau}P(t+\tau)\frac{\tilde{G}(t+\tau)}{\tilde{a}(t+\tau)}a(t,\tau)d\tau\right] e^{-rt}dt &= \\
\int_0^\infty \int_0^\infty e^{-r(t+\tau)}P(t+\tau)\frac{\tilde{G}(t+\tau)}{\tilde{a}(t+\tau)}a(t,\tau)dtd\tau &= \int_0^\infty \int_\tau^\infty e^{-rx}P(x)\frac{\tilde{G}(x)}{\tilde{a}(x)}a(x-\tau,\tau)dxd\tau = \\
\int_0^\infty \int_0^x e^{-rx}P(x)\frac{\tilde{G}(x)}{\tilde{a}(x)}a(x-\tau,\tau)d\tau dx &= \int_0^\infty e^{-rt}\int_0^t P(t)\frac{\tilde{G}(t)}{\tilde{a}(t)}a(t-\tau,\tau)d\tau dt
\end{aligned}$$

Thus, the objective function can be written as:

$$\begin{aligned}
&-\int_0^\infty \left[Q(A(t)) - R(a(t),b(t))\right] e^{-rt}dt + \int_0^\infty P(t)\frac{\tilde{G}(t)}{\tilde{a}(t)}\left[\int_0^\infty a(t,\tau)d\tau - \int_0^t a(t-\tau,\tau)d\tau\right] e^{-rt}dt \\
= &-\int_0^\infty \left[Q(A(t)) - R(a(t),b(t))\right] e^{-rt}dt + \int_0^\infty P(t)\tilde{G}(t)e^{-rt}dt
\end{aligned}$$

The problem is now the same as the one in the CFM section, with $\tilde{a}(t)$ instead of $a(t)$ and $\tilde{G}(t)$ instead of $G(t)$. Thus, by Lemma 1, the LCE is efficient under the condition shown. ■

5 Conclusion

We have set out a general framework to discuss carbon sequestration programs when different alternatives are available and each of them yields sequestration benefits far into the future and at varying rates. We have mainly focused our discussion on reforestations, since trees grow for a long time, varying the rate of growth as they grow older, and different types of species yield completely different per hectare sequestration rates. However, our framework is general enough to cover soil carbon sequestration practices or any other carbon sequestration practice where the characteristics just described are relevant. Since we have shown that the steady-state is of limited interest in our framework, the main challenge from a technical point of view has been to analyze the optimal sequestration path in a model with three control variables and three state equations, one of them of the Volterra type.

We have shown that if the carbon sequestration program is implemented using the Carbon Flow Method (or any of the other three methods analyzed as long as the additional conditions shown are met) the landowner and the Social Planner would choose the same species to reforest the same area at any moment in time. We have then shown that, except at the steady-state where no reforestations take place, the Social Planner (and the landowner) will continuously change the species used for the reforestations. Thus, postulating that only one type of species is available implies missing an important part of the story. In addition, since the trend is to use slower and slower growing species as the land available for reforestations becomes scarcer, the last reforestations will be done with very slow growing species so that carbon sequestration programs will continue to have an impact on climate change long after the last reforestations are done.

Finally, since species with longer rotations tend to have larger biodiversity and/or scenic values (Caparrós *et al.*, 2009), an indirect policy implication of our model is that all the land available should not be used to plant fast growing species to meet immediate targets (such as those set up in the Kyoto Protocol), since even without taking into account biodiversity and/or scenic values the optimal policy would be to use slower and slower growing species as land becomes scarcer and carbon builds up in the atmosphere. However, integrating biodiversity values explicitly in the analysis is left for future research. The other relevant caveat of our model is that we do not allow the type of forest to change in the future. Feng *et al.* (2002) did show in their model that foresting first and deforesting afterwards is not an optimal strategy, but changing the type of forest by cutting the old one and replacing it by a different type of forest might be optimal. We also leave this issue for future research.

References

[1] Bakke, V.L., 1974. A maximum principle for an optimal control problem with integral constraints. *Journal of Optimization Theory and Applications,* 13(1): 32-55.

[2] Calvo-Calzada, E. and Goetz, R.U., 2001. Using distributed optimal control in economics: a numerical approach based on the finite element method. *Optimal Control Application and Methods* 22: 231-249.

[3] Caparrós, A., 2009. Delayed carbon sequestration and rising carbon prices. *Climatic Change* 96: 421-441.

[4] Caparrós, A. and Jacquemont, F., 2003. Conflicts between biodiversity and carbon offset programs: economic and legal implications. *Ecological Economics*, 46: 143-157.

[5] Caparrós, A, Cerdá, E., Ovando, P. and Campos, P., 2009. Carbon sequestration with reforestations and biodiversity-scenic values. *Environmental and Resource Economics* (available on-line, DOI 10.1007/s10640-009-9305-5).

[6] Costa-Duarte, C., Cunha-e-Sá, M.A. and Rosa, R., 2006. *Forest vintages and carbon sequestration.* FEUNL Working Paper Series No. 482.

[7] Englin, J. and Callaway, J.M., 1993. Global Climate Change and Optimal Forest Management. *Natural Resource Modeling* 7 (3): 191-202.

[8] Feng, H., Zhao, J. and Kling, C.L., 2002. The time path and implementation of carbon sequestration. *American Journal of Agricultural Economics,* 84(1): 134-149.

[9] Intergovernmental Panel on Climate Change (IPCC), 2007. *Climate Change 2007: Synthesis Report.* IPCC, Geneva.

[10] Kamien, M.I. and Muller, E., 1976. Optimal control with integral state equations. *Review of Economic Studies* 43, 469-473.

[11] Lubowski, R.N., Plantinga, A.J. and Stavins, R.N., 2006. Land-use change and carbon sinks: Econometric estimation of the carbon sequestration supply function. *Journal of Environmental Economics and Management* 51: 135-152.

[12] Moura-Costa, P. and Wilson, C., 2000. An equivalence factor between CO_2 avoided emissions and sequestration – description and applications in forestry. *Mitigation and Adaptation Strategies for Global Change,* 5 (1): 51–60.

[13] Muller, E. and Peles, Y.C., 1990. Optimal dynamic durability. *Journal of Economic Dynamics and Control* 14: 709-719.

[14] National Forest Genetics Laboratory (NFGEL) and Genetic Resources Conservation Program (GRCP), 2006. *Why is Genetic Diversity Important? Why We Care About Genetics Vol.1.* (www.grcp.ucdavis.edu/projects/GeneticFactsheets/Vol_01_print.pdf, downloaded 4/11/2008).

[15] Olschewski, R. and Benitez, P.C., 2005. Secondary forests as temporary carbon sinks? The economic impact of accounting methods on reforestation projects in the tropics. *Ecological Economics,* 55: 380-394.

[16] Ragot, L. and Schubert, K., 2008. The optimal carbon sequestration in agricultural soils: Do the dynamics of the physical process matter? *Journal of Economic Dynamics and Control* 32: 3847-3865.

[17] Richards, K.R., Sampson, R.N. and Brown, S., 2006. *Agricultural & Forestlands: U.S. Carbon Policy Strategies*. PEW Center, Washington.

[18] Salo, S. and Tahvonen, O., 2002. On Equilibrium Cycles and Normal Forests in Optimal Harvesting of Tree Vintages. *Journal of Environmental Economics and Managent* 44: 1-22.

[19] Schmalensee, R., 1979. Market structure, durability and quality: A selective survey. *Economic Inquiry* 17, 177-196.

[20] Sohngen, B. and Mendelsohn, R., 2003. An optimal control model of forest carbon sequestration. *American Journal of Agricultural Economics,* 85(2): 448-457.

[21] Stavins, R.N., 1999. The costs of carbon sequestration: a revealed-preference approach. *American Economic Review* 89 (4): 994–1009.

[22] Tavoni, M., Sohngen, B. and Bosetti, V., 2007. Forestry and the Carbon Market Response to Stabilize Climate. *Energy Policy* 35: 5346-5353.

[23] van Kooten, G.C., Binkley, C.S. and Delcourt, G., 1995. Effects of Carbon Taxes and Subsidies on Optimal Forest Rotation Age and Supply of Carbon Services. *American Journal of Agricultural Economics*, 77: 365-374.

[24] van Kooten, G.C., 2000. Economic dynamics of tree planting for carbon uptake on marginal agricultural lands. *Canadian Journal of Agricultural Economics,* 48: 51-65.

[25] van't Veld, K. and Plantinga, A., 2005. Carbon sequestration or abatement? The effect of rising carbon prices on the optimal portfolio of greenhouse-gas mitigation strategies. *Journal of Environmental Economics and Management,* 50: 59-81.

[26] Vinokurov,V.R., 1969. Optimal control processes described by integral equations I. *SIAM Journal Control* 7(2): 324-336.

[27] Watson, R.T., Noble, I.R., Bolin, B., Ravindranath, N.H., Verardo, D.J. and Dokken, D.J (eds.), 2000. *IPCC Special Report on Land Use, Land-Use Change, and Forestry.* Cambridge University Press, Cambridge.

[28] Xabadia, A., Goetz, R.U. and Zilberman, D., 2006. Control of accumulating stock pollution by heterogeneous producers. *Journal of Economic Dynamics and Control* 30: 1105-1130.

APPENDIX: Steady-state

To derive the steady-state values, we focus on the particular growth function $g^E(t, b(s), s) = e^{-b(s)(t-s)}G^0 b(s)^\beta$, where $G^0 b(s)^\beta$ is the initial growth of the species chosen at time s and $b(s)$ is the decay rate of this initial growth, with $G^0 > 0$ (remark that since $b(s)$ is not constant the resulting integral for G continues to be of the Volterra type). This allows us to simplify the analysis but implies are much more restrictive set of growth functions than those used in the main text. The exponential function is a relatively good approximation of the growth of un-managed and even managed permanent forests (see van't Veld and Plantinga (2005)) but the main implicit assumption of this functional form is that there is a monotonic relationship between the growth rate and the ultimate volume, $M(b(s))$, reached by the species. If $\beta > (<)\ 1$ the ultimate volume is monotonically increasing (decreasing) in the growth rate. Unfortunately it is unclear if such a relationship exists in reality. Nevertheless, accepting this limitation, the Appendix shows that analyzing the steady-state, as is common in vintage models in economics, is of limited interest in our framework since no reforestations take place in the steady-state and therefore no species are chosen at all.

We start by noting that at any point in time there will be an "average" rate at which the total growth will be reduced equal to:

$$\rho(t) = \frac{\int_0^t b(s)a(s)G^0 b(s)^\beta e^{b(s)(s-t)}ds}{\int_0^t a(s)G^0 b(s)^\beta e^{b(s)(s-t)}ds} = \frac{\int_0^t b(s)a(s)G^0 b(s)^\beta e^{b(s)(s-t)}ds}{G(t)}$$

Thus, we can write (3) as:

$$\dot{G}(t) = a(t)G^0 b(t)^\beta - \rho(t)G(t) \tag{36}$$

In the steady-state the weighted average $\rho(t)$ is:

$$\rho(t) = \frac{1}{G(t)}\left[\int_0^{t^*} b(s)a(s)G^0 b(t)^\beta e^{b(s)(s-t)}ds + b^* \int_{t^*}^t a^* G^0 b^{*\beta} e^{b^*(s-t)}ds\right] \tag{37}$$

where the (*) variables are steady-state values so that $b(t) = b^*$ and $a(t) = a^*$ for all $t > t^*$. Now, letting t approach infinity the first term inside the brackets in (37) approaches zero and the second term approaches $b^*G(t)$ so that $\lim_{t\to\infty} \rho(t) = b^*$. Thus, (36) becomes, at the steady-state:

$$G^* = a^* G^0 b^{*\beta-1} \tag{38}$$

Taking the time derivative of (11) and (12), passing to the steady-state and finally substituting $g(t, b(t), t)$, we obtain

$$\begin{aligned} \frac{Q_A(A^*) + rR_a}{G^0 b^{*\beta}} &= B_e(e^*) && (39) \\ rR_b &= a^* \beta G^0 b^{*\beta-1} B_e(e^*) && (40) \end{aligned}$$

The steady-state values are obtained from equations (38)-(40) and the following equations obtained, respectively, from equations (2), (4), and the combination of (5) and (8):

$$e^* = \sigma C^* \quad (41)$$

$$a^* = 0 \quad (42)$$

$$D_c(C^*) = \sigma B_e(e^*) \quad (43)$$

Equations (41) and (43) uniquely determine the optimal emissions and the optimal stock of carbon in the atmosphere and show that carbon sequestration practices play no role in shaping these steady-state values. In addition, equation (42) shows that, at the steady-state, no reforestations take place. This implies, in turn, that the sequestration (G^*) at the steady-state obtained from equation (38) is also zero; and that the optimal value for b^* (type of species used for reforestation at the steady-state) obtained from equation (40) has no economic meaning, since no reforestations take place. Equation (39) indicates that, at the steady state, the sum of the opportunity cost of land and the opportunity cost of the cost of reforesting one additional hectare (divided by the initial growth of the steady-state species) has to be equal to the marginal benefit from emissions.

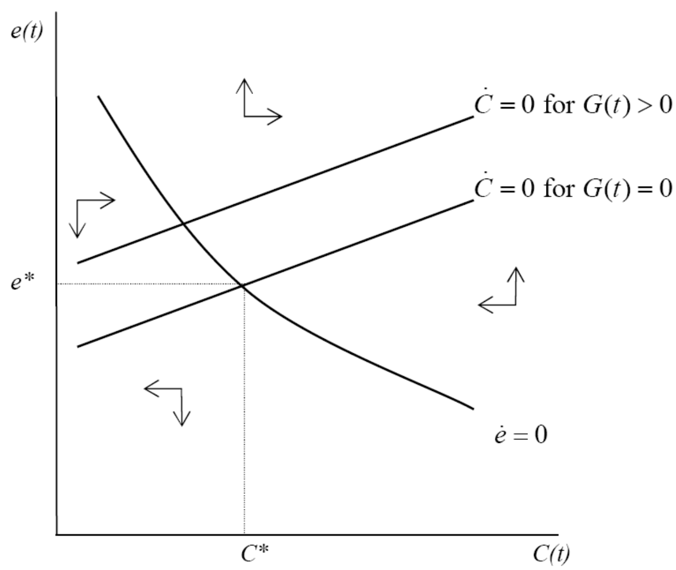

Figure 1. Phase diagram for carbon emission and stock.

Working Papers published in: Instituto de Políticas y Bienes Públicos (IPP), 2010

1. **Cruz Castro, L. & Sanz Menéndez, L.** Endogamia, Productividad y Carreras Académicas.

2. **Corrochano, D.** Guía Bibliográfica sobre Inmigración en España (1990-2009). Datos y Reflexiones sobre la Institucionalización de una Comunidad Académica.

3. **Golob, S.R**. Evolution or Revolution? Transitional Justice Culture Across Borders.

4. **Arias Aparicio, F.** Organización y Producción del Conocimiento Científico en los Organismos Públicos de Investigación Agraria: El Instituto Nacional de Investigación Agraria y Alimentaria (INIA).

5. **Moreno, L.** Welfare Mix, CSR and Social Citizenship.

6. **Martínez, C. & Rama, R.** The control and generation of technology in European food and beverage multinationals.

7. **Del Pino, E. & Colino, C.** National and Subnational Democracy in Spain: History, Models and Challenges.

8. **Closa, C.** Negotiating the Past: Claims for Recognition and Policies of Memory in the EU.

9. **Martínez, C., Cruz Castro, L. & Sanz Menéndez, L**. Convergencia y diversidad en los centros de I+D.

10. **Osuna, C., Cruz Castro, L. & Sanz Menéndez, L**. Knocking down some assumptions about the effects of evaluation systems on publications.

11. **Pavone, V., Goven, J. & Guarino, R.** From risk assessment to in-context trajectory evaluation: GMOs and their social implications.

12. **Pavone, V. & Arias, F.** Pre-Implantation Genetic Testing in Spain: beyond the geneticization thesis.

13. **Chinchilla-Rodríguez, Z., Corera-Álvarez, E., De Moya-Anegón, F. & Sanz-Menéndez**, L.(2010). Indicadores bibliométricos de España en el mundo 2008

14. **Jonkers, K. & Cruz-Castro, L.** The internasionalisation of public sector research through international joint laboratories.

15. **Encaoua, D., Guellec, D. & Martínez, C.** Sistemas de patentes para fomentar la innovación: lecciones de análisis económico.

16. **Oviedo, J. L., Campos, P. & Caparrós, A.** Simulated Exchange Value method: applying green national accounting to forest public recreation.

17. **Caparrós, A. & Péreau, J-C.** Coalition formation and bargaining power: Theory and application to international negotiations on public goods

Working Papers published in:
Instituto de Políticas y Bienes Públicos (IPP), 2010

18. **Caparrós, A. & Zilberman, D.** Optimal carbon sequestration path when different biological orphysical sequestration functions are available